From perception to action: an economic model of brain processes *

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This version: February 2009

Abstract

We build on evidence from neurobiology to model the process through which the brain maps evidence received from the outside world into decisions. This mechanism can be represented by a decision-threshold model. The sensory system encodes information in the form of cell-firing. Cell-firing is then measured against a threshold and an action is triggered depending on whether the threshold is surpassed. The decision system is responsible for modulating the threshold. We show that, for a large class of situations, the (constrained) optimal threshold is set in a way that existing beliefs are likely to be confirmed. We then derive behavioral implications of this theory. Our mechanism predicts: (i) belief anchoring (the order in which evidence is received affects both beliefs and choices); (ii) polarization (individuals with opposite priors may polarize their opinions when receiving mixed evidence); (iii) payoff-dependence of beliefs and (iv) belief disagreement (individuals with identical priors who receive the same evidence may end up with different posterior beliefs).

Keywords: Neuroeconomic theory, neurobiology, neuroscience, information processing, Bayesian learning.

*This paper is a major revision of the paper entitled "Reason, Emotion and Information Processing in the Brain". We thank C. Camerer, E. Dal Bo, A. Daughety, E. Fehr, J.J. Ganuza, D. Houser, A. Rangel, J. Reinganum, J. Schall, S. Scotchmer, R. Seamans, G. Tan and seminar participants at U. Carlos III, the behavioral conference at Fundación Areces, USC, U. of Guelph, UC Davis, UC San Diego, ASSA 2008, the conference on Neuroeconomics and the Law at USC, the Neuroeconomics conference at the Arne Ryde Foundation, the CSLP Symposium on Cognitive-Neuroscience and the Law at USC, UC Berkeley Haas Business School, ITAM and Vanderbilt University for helpful comments. Address for correspondence: Isabelle Brocas or Juan D. Carrillo, Department of Economics, University of Southern California, Los Angeles, CA 90089, <brocas@usc.edu> or <juandc@usc.edu>.

1 Introduction

Economic theory has traditionally been interested in the analysis of choices. In particular, the *processes* by which individuals reach decisions have been overlooked, mainly because we had little knowledge of the pathways going from perception to action. With the recent developments in techniques to measure brain activity, the neurobiology and neuroscience literatures have substantially improved our understanding of the *biological* mechanisms that transform sensory perceptions into voluntary actions. These results can now be incorporated into formal economic models of decision-making.[1]

The objective of this paper is to offer a brain-based model of information processing that builds on evidence from neurobiology. According to this evidence (extensively reviewed in section 2), there are five basic principles in the physiological mechanisms of information processing. *First*, neurons carry information from the sensory system to the decision system, using an imperfect encoding technology: the level of neuronal cell firing depends stochastically on the information obtained. *Second*, an action is triggered when the cell firing activity in favor of one alternative reaches a certain threshold. *Third*, neuronal activity responds to changes in payoffs and beliefs, that is, neurons compute approximately the 'expected value' associated to each alternative. *Fourth*, neurons also perform approximately 'Bayesian' inferences conditional on the data retained. *Fifth* and crucial for our purpose, the decision system has the ability to modify the triggering threshold. By acting on the threshold, the decision system affects how the evidence is interpreted.

Building on these premises, our analysis has three goals. First, we propose the first theoretical model capable of predicting decisions when the underlying mechanism that processes information exhibits the basic premises of neurobiology. Second, we characterize the decisions that result from this mechanism in environments with different levels of complexity. Third and most importantly, we analyze the behavioral implications of our theory for decision-making. In particular, we discuss behaviors that could not arise in a traditional learning model. We now review these three contributions.

In section 2, we represent the underlying mechanism that maps sensory perception into actions. To formalize the neurobiological principles described above, we start with a very simple, two-states (A and B), two-actions (a and b) model, where payoffs depend on the combination of action and state. The individual receives a signal from the outside world

[1] To our knowledge, Shefrin and Thaler (1988) was the first study to explicitly point out this possibility. However, few studies have followed this route.

which is encoded by the sensory system in the form of cell firing. The information stochastically depends on the state, with high cell firing being more likely in state A and low cell firing in state B. The decision system is an 'intelligent' threshold-based information processing system. Cell firing is measured against a threshold. If it is surpassed, action a is undertaken, otherwise action b is implemented. This model represents the underlying economical mechanism that transforms perception into action. Building on the previous evidence, we assume that the decision system selects the threshold that maximizes the expected payoff of the individual.[2] This representation is extended to complex and behaviorally more relevant environments that include many actions and states, and allow the decision system to set several thresholds. The model thus provides a common language for economists and neuroscientists interested in the study of decision processes.

In section 3, we characterize information processing and decision-making. In the basic setting with two states and two relevant actions, the threshold is optimally set in a way that beliefs are likely to be supported. That is, if the agent becomes more confident that the state is A, the threshold is decreased. Thus, the new threshold is more likely to be surpassed *whether the state is indeed A or not* and, as a result, the agent is more likely to take the action which is optimal in that state but suboptimal in the other (Proposition 1). The logic for this property is simple: as the likelihood of A increases, stronger contradictory information is required to reverse that belief. This result matches the findings obtained in the classical theory of organizations literature (Calvert (1985), Sah and Stiglitz (1986), Meyer (1991)) using related models. The contribution of the theory section is to extend the model to behaviorally relevant environments, where decisions are typically complex. We show that the result mentioned above generalizes in a number of dimensions. In particular, the basic property that thresholds are set in a way that beliefs are likely to be supported holds under mild technical conditions when there is a continuum of relevant actions (Proposition 2). The same conclusion also applies if the individual receives evidence sequentially and the decision-system re-optimizes the threshold during that process (Proposition 3). In fact, the ability to modify neuronal thresholds has a snowball effect on decision-making: a stronger belief towards one state implies a greater threshold modulation in its favor, therefore a higher probability that new information supports it, and so on. Finally, we also show that optimal thresholds are set in such a way that the processed information is most likely to be consistent with the action that yields highest payoff.

[2]Optimization in this context is a working assumption which may be defended on evolutionary grounds. It is obviously debatable. However, we feel that any other assumption would have been more ad-hoc and therefore even less satisfactory.

In section 4, we provide some behavioral implications of this theory. This is the third and possibly most fundamental contribution of the paper. Information processing in the brain has two major ingredients. On the one hand, the information received from the outside world and encoded by the sensory system is imperfectly correlated with the true state. This is typically equivalent to a standard learning environment where the signal is a distorted (noisy) report about the true state. On the other hand, the decision threshold mechanism itself filters out some information. In other words, the evidence received from the sensory system is *interpreted*. We analyze the effect of the interpretative feature of the mechanism, and focus on behaviors that would not emerge in a standard learning environment where the exact signals were processed. We obtain five implications. First, we show that the sequence in which signals are received affects both posterior beliefs and decisions (Implication 1). Indeed, given the threshold mechanism induces the individual to confirm his prior beliefs, a first piece of information may act as an anchor for subsequent learning experiences. Second, individuals with different prior beliefs who receive identical signals may move their beliefs farther apart (Implication 2). This is again due to the interpretative feature of the threshold mechanism. Suppose two individuals are willing to take two different actions following their prior beliefs, releasing mixed evidence will reinforce their priors and individuals will polarize both their beliefs and their actions. Third, individuals with identical priors but different payoffs will hold systematically different posterior beliefs (Implication 3). Indeed, individuals who feel differently about the losses incurred when the wrong action is undertaken will set different thresholds and therefore interpret the same evidence in different ways. Thus, preferences shape beliefs. Fourth, the optimal decision-threshold mechanism generates payoff-dependent posterior beliefs (Implication 4). As such, the preferences of an individual are best represented by an expected utility function where probabilities are payoff-dependent. The formulation is thus reminiscent of the rank-dependent expected utility and the security-potential/aspiration models. Fifth, cognitive limitations that result in setting an insufficient number of thresholds generates elimination strategies (Implication 5). More precisely, the decision-system disregards the most unlikely states and discriminates optimally among the remaining ones. It is important to note that each of these five implications are reminiscent of recurrent findings in social psychology. In particular, an abundant literature on confirmatory biases documents anchoring and polarization effects consistent with our two first implications. The relationship between preferences and beliefs has also been long noted. Therefore, our model provides a representation of the underlying neuronal mechanisms that generate some well known biases in a unified framework. Finally, in section 5 we draw a parallel between our model and two leading theories of decision-making in neuroscience, somatic marker and cognition control. Both argue that optimal decision-making is reached vía a biasing process that typically favors some actions. Our model provides a representation of such biasing process.

Our work departs substantially from the existing neuroeconomics literature. Current theories are largely interested in conflicts between brain systems. Behavioral anomalies are the result of an interplay between a rational and an automatic system.[3] Instead, we propose a model based on primitives that incorporates the *physiological constraints faced by the brain in the process of decision-making*. Since we focus on the physiological mechanisms behind the choice process, our paper is closer to the "physiological expected utility" theory developed by Glimcher et al. (2005).[4] There are also several areas outside economics that study a related problem, although from a rather different angle. Neurobiologists have worked on complex statistical algorithms that mimic what neurons in the brain do (see e.g. Brown et al. (1998)). Theoretical neuroscientists have constructed computational models of the brain based on the underlying biological mechanisms (see Dayan and Abbott (2005) for an introduction). Psychophysicists have developed a "signal-detection theory" to study the likelihood of finding a weak signal in a noisy environment (see Wickens (2002) for an introduction). Finally, there is a literature on neural networks and artificial intelligence which builds models inspired by the architecture of the brain in order to solve specific tasks like data processing or filtering (see Abdi (1994) for an overview).

The paper is organized as follows. In section 2, we present our model and review the neurobiology literature. In section 3, we characterize the optimal thresholds and subsequent decisions in environments with varying levels of complexity. In section 4, we develop several implications for choice under uncertainty. In section 5, we relate our results to recent theories in neuroscience. In section 6, we provide some concluding remarks. All the proofs and some supplementary material (extensions, robustness of the theory, and analytical examples) are relegated to the appendix.

[3]See e.g. Bernheim and Rangel (2004). Fudenberg and Levine (2006) and Brocas and Carrillo (2008b) follow Shefrin and Thaler (1988) in modelling the conflict in a slightly different way: different system have different temporal horizons. See also Zak (2004) or Camerer et al. (2005) for reviews.

[4]There is also a recent literature that explores the neurobiological foundations for social behavior, although it is almost exclusively experimental (see for e.g. Zak et al. (2004) on the effect of oxytocin on trust, Spitzer et al. (2007) for a study of the neural mechanism underlying social compliance and Fehr and Camerer (2007) for an overview).

2 Modelling brain processes

Consider a primitive individual (he) who chooses between staying in the cave and going hunting. On a dangerous day, the decision to go hunting can result in an injury. On a safe day, he may catch a prey and save his family from starvation. Before making the decision, the individual can take a look outside the cave. This conveys information about the hypothesis he should endorse: is it dangerous to leave the cave, or is it safe?

The brain uses a specific mechanism to analyze situations and make decisions. Neurons carry information from the sensory circuitry, where information about the outside world is received, to the decision-making circuitry, where the information is aggregated and interpreted. The objective of this section is to provide a parsimonious model able to represent the brain mechanisms underlying the decision process. We will first concentrate on a two-action discrimination task. This is the situation that has been documented most extensively in neurobiology. To simplify the exposition, we will use the previous example as an illustration. The model is generalized in section 2.5.

2.1 Building blocks of the theory

From an an abstract viewpoint, there are three ingredients necessary to make a decision.

□ *Environment.* It consists in the set of alternatives and states of the word. A distinctive feature of these elements is that the decision-maker cannot affect them in any way. In our example, the alternatives are two actions, a (stay in the cave) and b (go hunting) and the states of the world are hypotheses about the number of predators out there, A (dangerous day with many predators) and B (safe day with few predators). The action space is denoted by Γ with typical element γ, and the state space is denoted by S with typical element s.

□ *Preferences over outcomes.* They reflect how the individual feels about the consequences of his decision. In our example, there is an optimal action in each state: it is better to stay in the cave on a dangerous day and to go hunting on a safe day. When actions and states are not matched properly, the individual incurs losses (injury, starvation). This can be represented by a loss function $\tilde{l}(\gamma, s)$ where $\gamma = a, b$ and $s = A, B$, $\tilde{l}(a, A) = \tilde{l}(b, B) = 0$ and $\tilde{l}(\gamma, s) \leqslant 0$ otherwise. The individual must form a representation of the losses he might incur, and losses can be objective or subjective (e.g., individuals may differ in valuing the cost of an injury, or the benefit of food for the family). We will denote by $\mathcal{L}$ the complete description of outcomes. It is a map from $\Gamma \times S$ into $\mathbb{R}_{-}$. In our example, it is simply $\mathcal{L} = \{\tilde{l}(a|A), \tilde{l}(b|A), \tilde{l}(a|B), \tilde{l}(b|B)\}$.

□ *Information.* The individual makes inferences from the information available. There are two types of knowledge. First, before taking a look out of the cave, the individual has a sense of how likely each state is. He may use memories of previous episodes to make that assessment. This information can be summarized in a *prior belief probability* $p \in (0, 1)$ that the state is A. Despite its importance, we will disregard how priors are formed and simply claim that the individual is able to make a decision based on prior observations if he does not get to look outside the cave. Second, the individual receives a *signal* about the outside world which is collected by the sensory circuitry (visual and auditory cortices for instance). We now need to understand what is a signal in the brain.

2.2 Signals in the sensory system

The information received from the outside world is encoded and translated into neuronal activity. This activity represents the signal in the brain. We will capture it with the continuous variable $c \in [0, 1]$, where $c = 0$ corresponds to a perfectly safe perceived environment and $c = 1$ to an unambiguously dangerous one. The mechanism underlying the construction of c can be described as follows. Each neuron detects 'danger' or 'no danger'. The variable c is interpreted as the fraction of neurons that detect 'danger', and therefore favor the hypothesis that the state is A.[5] This formalization encompasses two cases: the image received from the outside world is fully informative but some neurons fire in the wrong direction, or the image is not fully informative and interpreted as such.

The existence of an entire range of c captures the stochastic variability in neuronal cell firing. The evidence from neurobiology in favor of this hypothesis is overwhelming. Even when exposed to the same stimuli, neurons do not always fire in the same way. Different neurons detect different features and sometimes compete (Ma et al. (2006), Nichols and Newsome (2002)). Stochastic variability also captures the metabolic costs associated to activity (Laughlin et al., 1998), and the existence of noise in the process (e.g., stochastic neurotransmitter release (Stevens, 2003)). Part of the variability can also be due to the context in which the image is received (e.g., naturalistic conditions (Simoncelli, 2003)). Finally, initial signals are aggregated within and across populations of neurons that detect different features which, again, results in noisy aggregation (Shadlen et al., 1996).

[5] Signals may also be received independently from different sensory systems indicating how likely a danger is. In that case, c can be seen as the aggregate of this information.

At the same time, perception is related to state. Formally, when the state is s, the likelihood of c is $f(c|s)$, with $F(c|s) = \int_0^c f(y|s)dy$ representing the probability of a cell firing activity not greater than c. To capture the idea that high cell firing (a majority of neurons carrying the signal danger) is more likely to occur when $s = A$ and low cell firing (a majority of neurons carrying the signal no danger) is more likely to occur when $s = B$, we use a standard Monotone Likelihood Ratio Property (MLRP):[6]

Assumption 1 (MLRP) $\frac{\partial}{\partial c}\left(\frac{f(c|B)}{f(c|A)}\right) < 0$ *for all* c. **(A1)**

In this paper, we do not discuss the origin of the function $f(\cdot|s)$. The particular functional form may be specific to the task, or specific to the individual, or to the individual for that task. The function may be shaped over time through repeated exposure. For our purpose, it is simply given.

2.3 Decision mechanism in the decision system

A decision is made on the basis of the overall knowledge of the problem. This knowledge consists in preferences $\mathcal{L}$ and information (the prior p and the signal c). A decision is then a map between knowledge and alternatives. Formally, it is a function $\mathcal{D}$ that assigns an action in Γ to any triple $(\mathcal{L}, p, c)$. The objective of this section is to characterize this map under the assumption that (i) decision-making in the brain is an economical process and (ii) its representation must be consistent with actual neuronal cell-firing.

□ *Decision-thresholds.* Neuronal thresholds and synaptic connections filter information. Depending on how high neuronal thresholds and/or how strong synaptic connections are, neuronal activity will be stopped or propagated along a given path and trigger a particular action. This mechanism is economical in that it requires minimal knowledge to reach a decision. In a classical study, Hanes and Schall (1996) use single cell recording to analyze the neural processes responsible for the duration and variability of reaction times in monkeys. The authors find that movements are initiated when *neural activity reaches a certain threshold activation level*, in a winner-takes-all type of contest.[7] This evidence suggests that the mechanism can be represented by a decision-threshold mechanism: it is as if there exists a threshold x such that action b is triggered when $c < x$, and action a is triggered when $c > x$. At the same time, it filters information out. In other words,

[6]MLRP implies: (i) $\frac{1-F(c|A)}{f(c|A)} > \frac{1-F(c|B)}{f(c|B)}$, (ii) $\frac{F(c|A)}{f(c|A)} < \frac{F(c|B)}{f(c|B)}$, and (iii) $F(c|A) < F(c|B)\ \forall\ c \in (0, 1)$.

[7]Nichols and Newsome (2002) provide a further analysis of the type of situations where information processing is likely to follow a winner-takes-all vs. a vector averaging pattern.

the mechanism provides an 'interpretation' of the information. The sensory system collects c and the decision system interprets it as $c < x$ or $c \geqslant x$. The decision system compares alternatives via this mechanism (see Shadlen et al. (1996), Platt and Glimcher (1999), Gold and Shadlen (2001) and Ditterich et al. (2003) for further evidence). Overall, decision-making can be represented by a decision-threshold mechanism of the form:

$$\mathcal{D}(\mathcal{L}, p, c) = \begin{cases} a & \text{if} \quad c \geqslant x \\ b & \text{if} \quad c < x \end{cases}$$

□ *Expected utility evaluation.* A series of single neuron recording experiments with primates have demonstrated that changes in beliefs and changes in payoff magnitudes are correlated with neuronal activity (Platt and Glimcher (1999), Padoa-Schioppa and Assad (2006, 2008)). Hence, neurons are computing approximately the "expected value" associated to each alternative (see also Roitman and Shadlen (2002) and Glimcher et al. (2005)). It implies that information about outcomes and about the environment will affect the propagation of information. For our model, this evidence supports the idea that thresholds are modulated to respond to changes in the environment. Interestingly, it also suggests that the environment affects how the brain values the different options. We can therefore represent the objective of the decision system as an expected payoff function. It also implies that the decision system will evaluate each alternative in a way compatible with expected utility theory. Formally, when $c < x$ taking the recommended action b is valued at:

$$L(\Pr(A|\, c < x, p), b) = \Pr(A|\, c < x, p))\ \tilde{l}(b|A) + (1 - \Pr(A|\, c < x))\, \tilde{l}(b|B)$$

and when $c \geqslant x$ taking the recommended action a is valued at:

$$L(\Pr(A|\, c \geqslant x), a) = \Pr(A|\, c \geqslant x)\ \tilde{l}(a|A) + (1 - \Pr(A|\, c \geqslant x))\, \tilde{l}(a|B)$$

□ *Bayesian inferences.* Another strand of the literature shows that neurons perform approximately Bayesian inferences, that is, compute posterior probabilities that a given state is correct conditional on the data retained. One of the early theories, the "Efficient Coding Hypothesis" postulates that neurons encode information as compactly as possible, so as to use resources efficiently (Barlow (2001), Simoncelli (2003)). This theory has recently led to a myriad of sophisticated statistical models that describe bayesian stochastic processing of information by neurons in visual, auditory and haptic perception tasks (see e.g. Schwartz and Simoncelli (2001), Ernst and Banks (2002), Körding and Wolpert (2004) and Ma et al. (2006)). Building on the work by Hanes and Shall (1996), Shadlen et al.

(1996) and Gold and Shadlen (2001) study a motion discrimination task, where monkeys must decide whether the net direction of dots that appear on a monitor is upward or downward. The authors develop a theory of how information is processed. Neurons "compute" approximately the log-likelihood ratio of the alternatives in order to determine which hypothesis should be supported by the evidence. Thus, according to this result, neurons incorporate the two major ingredients of bayesian theory: prior probabilities, and stochastic information processing (see also Deneve et al. (1999) for a numerical simulation model). In terms of our model, and taking this evidence together, the likelihood that the state is A when $c > x$, can be estimated at its Bayesian posterior belief:

$$\Pr(A|c \geqslant x) \equiv \overline{p}(x) = \frac{[1 - F(x|A)]p}{[1 - F(x|A)]p + [1 - F(x|B)](1 - p)} \tag{1}$$

Similarly, if $c < x$, then likelihood that the true state is A is:

$$\Pr(A|c < x) \equiv \underline{p}(x) = \frac{F(x|A)p}{F(x|A)p + F(x|B)(1 - p)} \tag{2}$$

A trivial implication of Bayesian updating is that, for any p and x, the belief about state A is revised upwards if x is surpassed ($\overline{p}(x) > p$) and downwards if x is not reached ($\underline{p}(x) < p$). This captures the idea that low cell firing is an imperfect indicator of state B and high cell firing is an imperfect indicator of state A. Also, for a given p, suppose that x is surpassed. Then, the posterior belief about state A is revised upwards more strongly if c is more tightly correlated with the true state (in a stochastic dominance sense). This is consistent with evidence obtained in neurobiology. Using a similar experiment as Shadlen et al. (1996), Ditterich et al. (2003) show that when the task is more difficult (fewer dots move in synchrony), monkeys make more mistakes.[8]

It is interesting to note that an outside observer may conclude the subject holds non-Bayesian beliefs when those are elicited. This is the case because updating is done through an internally consistent but constrained Bayesian process. The signal received c is not used to its full extent. Rather, it is interpreted and part of its content is filtered out: only the summary statistic $c < x$ or $c \geqslant x$ is retained.

2.4 Optimal modulation

As explained before, the decision-threshold represents the mechanism through which information from the sensory system is translated into decisions. There is ample evidence that

[8]Similar results have also been obtained with human subjects using fMRI studies. For instance, Heekeren et al. (2004) find that the mechanism by which the brain of a monkey computes perceptual decisions is also at work for humans and for more sophisticated choices, such as image recognition.

neuronal thresholds and synaptic connectivities are modulated, modifying the conditions under which information is propagated along a pathway. The effect of setting a threshold is to decide how to interpret the evidence. At the level of neurons, this can be achieved by a combination of tools. First, neurotransmitter releases can affect neuronal thresholds (see Mogenson (1987)). Neurons that carry signals of 'danger' will fire only if they receive a sufficiently strong cue about the existence of predators. If their threshold is increased, they are less likely to fire. This also means that they will pass over a 'danger' signal more infrequently, and some information will be lost in the process. Second, an interplay between inhibitory neurons and excitatory neurons can be used. Third, synapses filter the information that is just relevant to make a decision (by adjusting their strength, see Klyachko and Stevens (2006)). Fourth, information is directed along pathways that are receptive to specific features (Miller and Cohen, 2001). In terms of our model, this means that x can and will be adjusted. Intuitively, it is natural that a high level of cell-firing should be interpreted differently depending on the preferences of the individual as well as his prior belief. As such, modulation must be the mechanism used by the decision system to account for $\mathcal{L}$ and p.

Given the possibility of modulation, we want to construct a mechanism that sets the threshold in such a way that decision-making is optimized. As discussed in the introduction, this assumption is exploratory. However, we do not want to impose any further ad-hoc constraint on decision-making. Optimality requires two conditions. First, x must be such that, given the information filtered, the best of the two actions is taken. Formally, $L(\underline{p}(x), b) > L(\underline{p}(x), a)$ when $c < x$ and $L(\overline{p}(x), b) \leqslant L(\overline{p}(x), a)$ when $c \geqslant x$. However, note that many thresholds satisfy this property. Let $X(\mathcal{L}, p)$ be the set of all such thresholds. At equilibrium, the optimal threshold $x^*(\mathcal{L}, p)$ must be in that set.[9] Second, among those, the optimal threshold is such that the decision rule yields highest expected utility. Formally, it solves:

$$\arg\max_{x} \; V(x) = \Pr(c > x) L(\overline{p}(x), a) + \Pr(c < x) L(\underline{p}(x), b)$$

Overall, if modulation serves the purpose of improving decision making, the threshold will be set in such a way that these two conditions are satisfied.

[9]In terms of our model, Platt and Glimcher (1999) provides evidence for the fact that the decision system chooses a threshold which is, at least, close to optimal.

2.5 General representation

The previous representation can be easily extended to include arbitrarily many states (capturing different intensities of danger) and actions (capturing different distances traveled by the individual in his hunting endeavor). Suppose the decision system must discriminate between n alternatives from Γ, each indexed by i, and denote by $\mathcal{P}(S)$ the prior probability distribution over states. The decision process can be represented by a decision-threshold mechanism with $n-1$ thresholds:

$$\mathcal{D}(\mathcal{L}, \mathcal{P}(S), c) = \begin{cases} 1 & \text{if} \quad c \in [0, x_1) = A_1 \\ 2 & \text{if} \quad c \in [x_1, x_2) = A_2 \\ \ldots & \\ n & \text{if} \quad c \in [x_{n-1}, 1] = A_n \end{cases}$$

This can be implemented by combining several neuronal pathways, each one discriminating between two alternatives.[10] Alternative i's value is $L(\mathcal{P}(S|c \in A_i), i) = E_s[\tilde{l}(i|s)|c \in A_i]$. Let $X_i(\mathcal{L}, \mathcal{P}(S))$ be the set of thresholds x_i such that it is optimal to take action i between x_{i-1} and x_i and let $V(x_1, ..., x_{n-1}) = E_{A_i}[L(\mathcal{P}(S|c \in A_i), i)]$. Finally, denote by $x_i^*(\mathcal{L}, \mathcal{P}(S))$ the optimal threshold. We obtain the following representation.

Representation. In a generic n-action discrimination task, the decision is represented by the following decision-threshold mechanism:

$$\mathcal{D}(\mathcal{L}, \mathcal{P}(S), c) = \begin{cases} 1 & \text{if} \quad c \in \Big[0, x_1^*(\mathcal{L}, \mathcal{P}(S))\Big) \\ 2 & \text{if} \quad c \in \Big[x_1^*(\mathcal{L}, \mathcal{P}(S)), x_2^*(\mathcal{L}, \mathcal{P}(S))\Big) \\ \ldots & \\ n & \text{if} \quad c \in \Big[x_{n-1}^*(\mathcal{L}, \mathcal{P}(S)), 1\Big] \end{cases}$$

where (i) $x_i^*(\mathcal{L}, \mathcal{P}(S)) \in X_i(\mathcal{L}, \mathcal{P}(S))$ for all i.
(ii) $x_i^*(\mathcal{L}, \mathcal{P}(S)) = \text{argmax}_{x_i \in X_i(\mathcal{L}, \mathcal{P}(S))} V(x_1, ..., x_{n-1})$ for all i.

Last, we conjecture that as the number of alternatives grow, the brain may fail to implement this mechanism. To study that possibility, we introduce the following definitions.

Definition 1 *An action is "relevant" if it is optimal to take it for some beliefs.*

Given this definition, if the decision-system can discriminate between all the relevant actions, it is optimal to do so.

[10]Determining the particular architecture of those pathways would be speculative. Existing neuronal cell recording research has focused almost exclusively on discrimination between two alternatives.

Definition 2 *A "cognitive process" is able to discriminate between all relevant actions. By contrast, an "affective process" must neglect some relevant actions.*

Naturally, we will use the same representation for both types of processes as the representation does not hinge on the relevance of the alternative. This is important. In our framework, an affective process does not make mistakes in a presupposed way. It simply cannot discriminate between all options, and therefore acts as if it faced tighter constraints than the cognitive process.

3 Optimal thresholds and action selection

In this section we characterize the optimal thresholds and subsequent decisions under different assumptions about the number of alternatives and states. To isolate the effect of each assumption, we study each case separately.

3.1 Choosing between many actions

Suppose there are only two states $S = \{A, B\}$ and a continuum of actions $\Gamma = [0, 1]$, where lower values of γ denote going farther away from the cave to hunt. The preferences over outcomes are captured by the following loss functions:

$$\tilde{l}(\gamma, A) = \pi_A \, l(\gamma - 1), \qquad \tilde{l}(\gamma, B) = \pi_B \, l(\gamma - 0)$$

where $l(z) = l(-z)$ for all z, $l'(z) < 0$ for all $z > 0$, and $\pi_s > 0$. Thus, the individual should go far away ($\gamma = 0$) on safe days ($s = B$), and stay close to the cave ($\gamma = 1$) on dangerous days ($s = A$). Note that π_s captures the marginal cost of taking a wrong action when the state is s. Therefore, $\pi_A > \pi_B$ means that hunting on a dangerous day is more costly than staying in the cave on a safe day (e.g., the probability of injury may be greater than that of starvation).

Given a posterior belief μ, the expected payoff of taking action γ is:

$$L(\mu, \gamma) = \mu \left[\pi_A \, l(\gamma - 1) \right] + (1 - \mu) \left[\pi_B \, l(\gamma) \right] \tag{3}$$

Suppose that $l(z)$ is weakly convex on both sides of its bliss point $z = 0$, $l''(z) \geqslant 0$ for all z, so that departures from the optimal action are decreasingly costly (a special case being the linear loss function $l(z) = -|z|$). In that case, $L(\mu, \gamma)$ is weakly convex in γ and differentiable on $(0,1)$, so corner solutions are optimal. Denoting by $\gamma^*(\mu) = \arg\max_\gamma \; L(\mu,\gamma)$, using (3) and given $l(0) > l(1)$, we have:

$$\gamma^*(\mu) = \begin{cases} 1 & \text{if} \quad L(\mu;1) \geqslant L(\mu;0) \;\Leftrightarrow\; \mu \geqslant p^* \equiv \pi_B/(\pi_A+\pi_B) \\ 0 & \text{if} \quad L(\mu;1) < L(\mu;0) \;\Leftrightarrow\; \mu < p^* \equiv \pi_B/(\pi_A+\pi_B) \end{cases}$$

Notice that $dp^*/d\pi_A < 0$ and $dp^*/d\pi_B > 0$: if the marginal cost of an incorrect action in a given state increases, then the individual is more willing to take the action optimal in that state even at the increased risk of erring in the other state. In our example, as predators become smarter and more dangerous, the individual is more likely to decide to stay in the cave, even on days that are apparently safe.

Suppose now that $l(z)$ is strictly concave , $l''(z) < 0$ for all z, so that departures from the optimal action are increasingly costly. Denote by $\gamma^{**}(\mu) = \arg\max_\gamma \; L(\mu,\gamma)$. Taking the first-order condition in (3), we have:

$$\frac{\pi_B}{\pi_A}\,\frac{l'(\gamma^{**}(\mu))}{l'(1-\gamma^{**}(\mu))} = \frac{\mu}{1-\mu} \tag{4}$$

Note that $\left.\frac{\partial L(\mu,\gamma)}{\partial\gamma}\right|_{\gamma=0} = -\pi_A\,\mu\,l'(1) > 0$, $\left.\frac{\partial L(\mu,\gamma)}{\partial\gamma}\right|_{\gamma=1} = \pi_B\,(1-\mu)l'(1) < 0$, and $\frac{\partial^2 L(\mu,\gamma)}{\partial\gamma^2} < 0$. This means that the optimal choice is always interior ($\gamma^{**}(\mu) \in (0,1)$) and that there is a continuum of relevant actions. Our first lemma summarizes these findings.

Lemma 1 *Suppose $S = \{A, B\}$ and $\Gamma = [0,1]$. If $l''(z) \geqslant 0$, there are only two relevant actions, $\gamma^*(\mu) = 1$ if $\mu > p^*$ and $\gamma^*(\mu) = 0$ otherwise. If $l''(z) < 0$, all $\gamma \in \Gamma$ are relevant.*

We can now characterize the optimal threshold mechanism in both situations.

3.2 Optimal threshold modulation when the loss function is convex

When $l''(z) \geqslant 0$, the problem boils down to a simple discrimination task with two states and two relevant actions. Applying our representation, the value function is:

$$\begin{aligned} V(x) &= \Pr(c > x)\,L(\overline{p}(x),1) + \Pr(c < x)\,L(\underline{p}(x),0) \\ &= p\,\pi_A\Big[(1-F(x|A))l(0) + F(x|A)l(1)\Big] + (1-p)\,\pi_B\Big[(1-F(x|B))l(1) + F(x|B)l(0)\Big] \end{aligned} \tag{5}$$

The optimal threshold is $x^* = \arg\max_x \; V(x)$, which leads to the following result.

Proposition 1 *When $l''(z) \geqslant 0$, the optimal process (cognitive or affective) involves a single decision threshold. This optimal threshold x^* is given by:*

$$\frac{f(x^*|B)}{f(x^*|A)} = \frac{p}{1-p}\,\frac{\pi_A}{\pi_B} \tag{6}$$

which, in particular, implies that $dx^*/dp < 0$ and $dx^*/d\mathcal{L}^* < 0$ where $\mathcal{L}^* = \pi_A/\pi_B$.

The result has three implications. First and foremost, the threshold is set in such a way that existing beliefs are likely to be confirmed. To see this, consider a symmetric situation with a prior belief $p > 1/2$. Setting a high threshold is not efficient: whether it is surpassed or not, the individual will still think that A is the most likely state. Instead, setting a low threshold is optimal. If it is surpassed, the individual will slightly increase his confidence in state A. If it is not reached, he will become convinced that the state is B. The individual will end up taking different actions depending on the signal and, in both cases, he will be confident about his choice. Overall, the optimal threshold balances the belief in favor of A conditional on the threshold being surpassed and the belief in favor of B conditional on the threshold not being reached. In order to achieve this balance, the threshold should be low whenever A is a priori more probable than B and high otherwise.[11]

Second, the optimal threshold is also affected by the relative payoffs. If the loss of taking the wrong action is higher in state A than in state B, then the threshold decreases. As a result, it will be surpassed with higher probability, whether the true state happens to be A or B. In equilibrium, the most costly mistakes are most likely to be avoided.

Third, the choice is efficient in the sense that the individual would not gain any extra utility if he could observe the exact level of cell firing c. Formally and using (6):

$$\Pr(A \,|\, c = x^*) = \frac{p\, f(x^*|A)}{p\, f(x^*|A) + (1-p)\, f(x^*|B)} = \frac{\pi_B}{\pi_A + \pi_B}$$

This means that, $\gamma^* = 1$ dominates $\gamma^* = 0$ *for all* $c > x^*$ and $\gamma^* = 0$ dominates $\gamma^* = 1$ *for all* $c < x^*$ which, in turn, implies that learning whether x^* is surpassed or not is sufficient for the purpose of determining which action to take. The result hinges on the weak convexity of the loss function and it is worth thinking under which circumstances that condition is met. Convexity reflects the fact that marginal departures from the ideal

[11] As a technical point, **(A1)** ensures uniqueness of maximum. If, instead, we imposed a (weaker) first-order stochastic dominance assumption ($F(c|B) > F(c|A)$ for all c), x^* would not necessarily be unique. However, since $V(x;p)$ is submodular, the monotonic relation between threshold and likelihood of state, $dx^*/dp < 0$, would be preserved in all the local maxima (we thank Guofu Tan for pointing this out).

choice are the most costly ones. Therefore, it is suitable to model environments where life-threatening events occur as soon as the optimal action is not taken ("basic" or "primitive" environments).

Overall, the main message of Proposition 1 is that a given piece of evidence is more likely to be interpreted as 'danger' if the individual a priori believes it is a dangerous day, and if losses of suboptimal actions are higher on dangerous days.

Finally, notice that Proposition 1 is reminiscent of Calvert (1985), Sah and Stiglitz (1986) and Meyer (1991). Besides the obvious differences in interpretations, the only (minor) formal difference is that those papers assume a two-action space whereas we consider a continuum of actions and show in section 3.1 that only two are 'relevant' when the loss function is convex. The theoretical contribution of the paper, however, is to determine the optimal thresholds in more complex situations such as concave losses and a dynamic setting. We turn to this analysis in the next sections.

3.3 Optimal threshold modulation when the loss function is concave

Suppose now that $l''(z) < 0$. As shown in section 3.1, there is a continuum of relevant actions, so a 'cognitive' process involves a continuum of thresholds. Because this may be difficult to implement in the brain, we determine optimal decisions when an 'affective' process is used instead. We assume that the decision system can only differentiate between two actions, but we do not presuppose which ones these are (the analysis can be easily extended to any finite set of actions). Under this affective process, the decision system sets one decision-threshold x. If it is surpassed, the updated belief is $\overline{p}(x)$. If it is not reached, the updated belief is $\underline{p}(x)$. Given (1), (2) and (4), the optimal actions satisfy:

$$\frac{\pi_B\, l'(\gamma^{**}(\overline{p}(x)))}{\pi_A\, l'(1-\gamma^{**}(\overline{p}(x)))} = \frac{\overline{p}(x)}{1-\overline{p}(x)} = \frac{p}{1-p}\,\frac{1-F(x|A)}{1-F(x|B)} \tag{7}$$

$$\frac{\pi_B\, l'(\gamma^{**}(\underline{p}(x)))}{\pi_A\, l'(1-\gamma^{**}(\underline{p}(x)))} = \frac{\underline{p}(x)}{1-\underline{p}(x)} = \frac{p}{1-p}\,\frac{F(x|A)}{F(x|B)} \tag{8}$$

where $\gamma^{**}(\overline{p}(x)) > \gamma^{**}(\underline{p}(x))$. Differentiating (7) and (8) and using **(A1)**, we obtain:

$$\frac{d\gamma^{**}(\overline{p}(x))}{dx} > 0 \quad \text{and} \quad \frac{d\gamma^{**}(\underline{p}(x))}{dx} > 0$$

An increase in the threshold always induces the individual to choose a higher action: if it is surpassed, then the evidence in favor of A is stronger whereas if is not reached, then the evidence in favor of B is weaker. In both cases, higher actions follow. The new value function is identical to (5) except for the actions selected in either case. Formally:

$$V(x) = \Pr(c > x)\, L(\overline{p}(x), \gamma^{**}(\overline{p}(x))) + \Pr(c < x)\, L(\underline{p}(x), \gamma^{**}(\underline{p}(x))) \tag{9}$$

The optimal threshold discriminates optimally between the two actions. To find this threshold, we first need to introduce a strengthened version of MLRP.

Assumption 2 (s-MLRP) *The probability distributions satisfy:*[12]

$$(i)\ \frac{f(c|B)}{1-F(c|B)} > \frac{f(c|A)}{1-F(c|A)},\ (ii)\ \left(\frac{f(c|B)}{f(c|A)}\frac{1-F(c|A)}{1-F(c|B)}\right)' \leqslant 0,\ (iii)\ \left(\frac{f(c|B)}{f(c|A)}\frac{F(c|A)}{F(c|B)}\right)' \leqslant 0 \tag{A2}$$

Define the following function:

$$H(x) \equiv F(x|B) l(\gamma^{**}(\underline{p}(x))) + (1 - F(x|B)) l(\gamma^{**}(\overline{p}(x)))$$

We can now state our next result.

Proposition 2 *When $l''(z) < 0$, the optimal cognitive process requires a continuum of thresholds. An affective process that discriminates only between two actions sets a threshold x^{**} and chooses actions $\gamma^{**}(\overline{p}(x^{**}))$ or $\gamma^{**}(\underline{p}(x^{**}))$. The optimal threshold satisfies:*

$$\frac{f(x^{**}|B)}{f(x^{**}|A)} = \frac{p}{1-p}\,\frac{\pi_A}{\pi_B}\,\frac{l(1 - \gamma^{**}(\overline{p}(x^{**}))) - l(1 - \gamma^{**}(\underline{p}(x^{**})))}{l(\gamma^{**}(\underline{p}(x^{**}))) - l(\gamma^{**}(\overline{p}(x^{**})))} \tag{10}$$

*It is unique and such that $dx^{**}/dp < 0$ if $dH(x^{**})/dx > 0$ and **(A2)** is satisfied.*

*Under **(A1)**, $dH(x^{**})/dx > 0$ guarantees $dx^{**}/dp < 0$ in every locally optimal threshold but not uniqueness. Last, $dx^{**}/d\mathcal{L}^* < 0$.*

When departures from the optimal action are increasingly costly, the quasi-concavity of the value function $V(x; p)$ is not guaranteed for generic values of $f(\cdot|A)$, $f(\cdot|B)$ and $l(\cdot)$. In fact, as x increases, two countervailing forces are at play. First and as before, the threshold is less likely to be surpassed and therefore more likely to induce the low action. Second, either outcome is a weaker indicator that the state is B. Therefore, the final action will be higher both when the threshold is surpassed and when it is not reached. Proposition 2 states that the qualitative conclusions of Proposition 1 extend to a concave loss function as long as the problem is well-behaved, that is, if $dH(x^{**})/dx > 0$. The interpretation of this condition is simple: starting from the optimal threshold, a marginal increase in

[12]Note that (i) and (ii) or (i) and (iii) in **(A2)** imply **(A1)**, but the converse is not true.

x increases the payoff of the individual if and only if the state is B. In other words, as x increases, the direct effect of increasing the likelihood of choosing the low action must dominate the indirect effect of choosing relatively higher actions. In Appendix 2, we show that this condition is automatically satisfied when payoffs are quadratic ($l(z) = \alpha - \beta z^2$) and $\pi_A = \pi_B$. We also provide a complete characterization of the optimal threshold for that event, and an analytical solution of x^* and x^{**} in the linear and quadratic cases given specific functional forms for the distribution functions. Note also that, as in Proposition 1, if the loss of taking the wrong action in a given state is increased, the individual will modify the threshold so as to favor that action.

Finally, it is important to realize that this affective process *does not* prevent the individual from taking any action in Γ. Decision-making is ex-post binary (one threshold implies only two possible actions) but ex-ante continuous (x can be set to reach any action). The result is then consistent with the experimental evidence according to which individuals can and often do report beliefs that vary continuously. Indeed, the ability to fine tune the threshold as a function of either past experience or changes in the experimental conditions results in continuous changes in expected beliefs, both when the thresholds are surpassed and when they are not met.

3.4 The dynamics of look-ups

In this section, we go back to the case of a convex loss function (hence, only two relevant actions) and assume that the individual sequentially obtains two signals regarding the environment. We impose the following assumptions on the representation of the dynamic process: (i) the process allows for threshold re-optimization between look-ups; (ii) it has memory, which means that the prior belief before the second look-up is simply the posterior belief after the first look-up; and (iii) it is forward-looking so that threshold modulation at each date takes into account future learning opportunities.

The rationale for these assumptions is the following. First, the dynamic setting is interesting only under re-optimization between look-ups, otherwise it boils down to one look-up and a more accurate information. Property (i) guarantees that we are in this scenario. Second, we do not want to presuppose the existence of any exogenous loss of information between look-ups. Given property (ii), all the information collected and processed after a look-up is retained. Third, we want to concentrate on 'intelligent processes' that operate under constraints, rather than impose ad-hoc limitations for contingent planning.

Property (iii) ensures the process optimizes at each period while anticipating future re-optimization. Needless to say, all these assumptions are exploratory.

The dynamic choice can be modeled as the following two-stage problem. The individual initially holds a prior belief p. At stage 1, the sensory system collects c_1. Given no immediate decision is required, the action at date 1 consists in summarizing the information received from the sensory system. In other words, an action at stage 1 is simply a report on the signal c_1. Information is interpreted using a cognitive or affective process, and the belief is updated. At stage 2, the sensory system collects c_2. Information is again interpreted using a cognitive or affective process and its recommendation is implemented. We assume that c_t is independently drawn from distribution $F_t(c_t|s)$ with $t \in \{1, 2\}$. Distributions may be different across stages but $\frac{f_t(c|B)}{f_t(c|A)}$ satisfies **(A1)** for all t.[13]

Again, we concentrate on optimal modulation and extend our earlier representation. The choice problem can be represented by a sequence of decision-threshold mechanisms. Given (iii), those mechanisms obey backward induction. At date 2, there are only two relevant actions, so whether the process is cognitive or affective, it sets one threshold and optimally discriminates between actions 0 and 1. We denote by x^* the optimal threshold, since it is identical to that described in section 3.2. At date 1, a cognitive process requires a continuum of thresholds (just like in the static case with a concave loss function described in section 3.3). An affective process that can discriminate only between two alternatives will set one forward looking decision-threshold that we denote by y^*. If it is surpassed, the posterior becomes $\overline{p}(y^*)$ and the optimal second stage threshold is $x^*(\overline{p}(y^*))$. If it is not reached, the posterior becomes $\underline{p}(y^*)$ and the optimal second stage threshold is $x^*(\underline{p}(y^*))$. Second stage thresholds are obtained using (1), (2) and (6). They satisfy:

$$\frac{f_2(x^*(\overline{p}(y))|B)}{f_2(x^*(\overline{p}(y))|A)} = \frac{\overline{p}(y)}{1-\overline{p}(y)}\frac{\pi_A}{\pi_B} = \frac{p}{1-p}\frac{1-F_1(y|A)}{1-F_1(y|B)}\frac{\pi_A}{\pi_B} \tag{11}$$

$$\frac{f_2(x^*(\underline{p}(y))|B)}{f_2(x^*(\underline{p}(y))|A)} = \frac{\underline{p}(y)}{1-\underline{p}(y)}\frac{\pi_A}{\pi_B} = \frac{p}{1-p}\frac{F_1(y|A)}{F_1(y|B)}\frac{\pi_A}{\pi_B} \tag{12}$$

The optimal threshold in the first stage maximizes the following value function:

$$W(y) = \Pr(c_1 > y)\Big[V(x^*(\overline{p}(y))\Big] + \Pr(c_1 < y)\Big[V(x^*(\underline{p}(y))\Big] \tag{13}$$

The first term is the likelihood of surpassing a cutoff y, in which case the posterior becomes $\overline{p}(y)$, multiplied by the second-stage value function given this posterior (see (5)),

[13] The setting assumes costless signals. It could be easily extended to costly signals, in which case the individual would also have to choose the optimal amount of signals.

and under the anticipation of an optimal second-stage threshold $x^*(\overline{p}(y))$ (see (11)). The same logic applies to the second term. Notice that threshold y affects the utility of the individual only through its effect on the posterior belief. Define the following function:

$$J(y) \equiv F_1(y|B)\, F_2(x^*(\underline{p}(y))|B) + (1 - F_1(y|B))F_2(x^*(\overline{p}(y))|B)$$

We can now state our next result.

Proposition 3 *In a dynamic setting with $l''(z) \geqslant 0$, the optimal cognitive process requires a continuum of first stage thresholds. An affective process that discriminates between two alternatives in the first stage sets a threshold y^* that satisfies:*

$$\frac{f_1(y^*|B)}{f_1(y^*|A)} = \frac{p}{1-p}\,\frac{\pi_A}{\pi_B}\,\frac{F_2(x^*(\underline{p}(y^*))|A) - F_2(x^*(\overline{p}(y^*))|A)}{F_2(x^*(\underline{p}(y^*))|B) - F_2(x^*(\overline{p}(y^*))|B)} \tag{14}$$

*It is unique and such that $dy^*/dp < 0$ if $dJ(y^*)/dy > 0$ and **(A2)** is satisfied.*

*Under **(A1)**, $dJ(y^*)/dy > 0$ guarantees $dy^*/dp < 0$ in every locally optimal threshold but not uniqueness. Last $dy^*/d\mathcal{L}^* < 0$.*

Two-stage optimization problems are easily plagued by non-convexities in the overall maximand. Proposition 3 states that the qualitative conclusions of previous propositions are preserved in the dynamic version of the model if a technical condition, $dJ(y^*)/dy > 0$, is satisfied. As before, the intuition relies on the balance between the likelihood of the information and its impact: the individual must sacrifice either quality or probability of obtaining information, and quality is relatively more valuable for the less favored state. In fact, the two-stage model with decreasingly costly departures is technically similar to the one-stage model with increasingly costly departures. In particular, the same two effects operate when the threshold is increased. First, a direct effect: the new threshold is less likely to be surpassed. Second, an indirect effect: because surpassing a higher threshold is a stronger indicator of A and not reaching it is a weaker indicator of B, an increase in stage 1 threshold is always followed by a decrease in stage 2 threshold ($dx^*(\overline{p})/dy < 0$ and $dx^*(\underline{p})/dy < 0$). Condition $dJ(y^*)/dy > 0$ ensures that the direct effect dominates the indirect one. In Appendix 3 we show that the condition automatically holds for any first-period distribution that satisfies **(A2)** if the second-period distributions are linear and symmetric. The result also highlights the dynamic snowball effect of threshold modulation on decision-making: a stronger belief towards one state implies a greater modulation in its favor, therefore a higher probability that new information supports it, and so on. We now provide below a simple analytical example that illustrates the theory.

Example 1. Suppose the density functions are identical in both stages, symmetric and linear: $f_t(c|A) = 2c$ and $f_t(c|B) = f(1-c|A) = 2(1-c)$, $t \in \{1, 2\}$. Let $\pi_S = 1$. From (6) and (14) and after some algebra, the optimal first and second stage thresholds are:

$$\left(\frac{1-y^*}{y^*}\right)^2 = \frac{p}{1-p} \;\Leftrightarrow\; y^*(p) = \frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}} \quad \text{and} \quad \frac{1-x^*}{x^*} = \frac{p}{1-p} \;\Leftrightarrow\; x^*(p) = 1-p$$

Notice that $x^*(p) \gtrless y^*(p) \gtrless 1/2$ for all $p \lessgtr 1/2$: for a given belief, the cutoff is always more extreme in the second than in the first stage. The intuition is that in the first stage the individual chooses the partition that conveys most information whereas in the second stage he chooses the partition that discriminates best among the two relevant alternatives. Finally, in Appendix 3, we compute for this example the expected utility difference between using a cognitive process (that employs a continuum of first stage thresholds) and an affective process (that employs only one)

One may wonder what happens as the number of information processing stages increases. Obviously, the final partition of beliefs becomes finer, which means that a greater number of posterior beliefs can be reached. More interestingly, with three or more stages, the thresholds at all but the last stage only affect the belief inherited at the following stage. Thus, we conjecture that the main properties of the thresholds emphasized in Propositions 1 and 3 should, under reasonable conditions, be preserved.[14]

3.5 Summary

The results of section 3 can be briefly summarized as follows. As in the theory of organizations literature, threshold processes are modulated in a way that existing beliefs are favored. Employing an affective rather than a cognitive process does not result in a utility loss if the environment is simple. That result, however, does not extend to environments that are behaviorally more relevant (increasingly costly losses, dynamic settings). By contrast, the existence, qualitative properties of optimal threshold modulation (variation as a function of priors) and comparative statics with respect to π_S are all preserved in those complex environments under mild technical conditions. The theory extends in a number of other directions. In Appendix 4, we show that all the comparative statics are also preserved if we consider a compact state space ($S \in [0, 1]$). In Appendix 5, we characterize the optimal threshold as a function of the accuracy of the information received to capture the difference between simple and complex activities. We show that optimal thresholds

[14]Unfortunately, we have not been able to solve this problem analytically.

are more sensitive to initial beliefs in complex than in simple situations. Finally, the comparative statics obtained in this section and related appendixes are also in line with the results of single cell recording activity reported in neuroscience experiments. In the next section, we will extensively use these general comparative statics results to discuss some implications for belief formation and choice under uncertainty.

4 Behavioral implications for choice under uncertainty

There are two main elements in our theory. First, the information encoded in the sensory system is stochastically correlated with the state. This is like a noisy report in a standard learning context. Second and most importantly, the decision-threshold mechanism results in a loss of information. In particular, all signals on the same side of the threshold are pooled and treated identically. This means that the evidence received from the sensory system is interpreted coarsely and imperfectly. In this section, we explore some behavioral implications of the model. We focus on the interpretative feature of the mechanism, and study behaviors that could not occur in a traditional environment where the exact signals were processed.

4.1 Confirmatory biases

In social psychology, a confirmatory bias is described as an error of inductive inference. It is the tendency of individuals to interpret evidence in a way that confirms their preconceived ideas about the world and avoid information that contradicts them (see Wason (1960) for a pioneering work and Nickerson (1998) for a review).[15] Our model offers a neurobiological mechanism for this bias. As developed in section 3, when the belief that the state is A becomes stronger, the individual sets a lower threshold. Evidence is then more likely to be interpreted as endorsing A and less likely to be interpreted as endorsing B, both if the true state is A and if the true state is B. This simple principle has some subtle implications.

Belief anchoring and the role of first impressions

Suppose the decision-maker has a flat prior about an issue and receives a first piece of evidence that he uses to build a belief. According to our theory, once a belief is anchored,

[15] As beautifully expressed by Leo Tolstoi, "the most difficult subjects can be explained to the most slow-witted man if he has not formed any idea of them already; but the simplest thing cannot be made clear to the most intelligent man if he is firmly persuaded that he knows already, without a shadow of doubt, what is laid before him." (The Kingdom of God is Within You, Chapter III).

it is likely to be reinforced when the decision-maker is exposed to new information. This yields the following implication.

Implication 1 *(Belief anchoring) The sequence in which signals are received affects the beliefs and actions of the individual.*

Consider the dynamic setting of section 3.4. Suppose for simplicity the environment is symmetric ($f(c|A) = f(1-c|B)$ and $\pi_A = \pi_B$) and the prior belief is $p = 1/2$, so that the first stage threshold is $y^*(1/2) = 1/2$. Consider the following symmetric signals $\underline{c}$ and $\overline{c}$, with $\overline{c} = 1 - \underline{c}$, $\underline{c} \in (x^*(\Pr[A|c > 1/2]), 1/2)$ and therefore $\overline{c} \in (1/2, x^*(\Pr[A|c < 1/2]))$. If $c_1 = \overline{c}$ and $c_2 = \underline{c}$, the thresholds at both stages are surpassed. The individual has a posterior belief $\Pr[A|c_1 > 1/2, c_2 > x^*(\Pr[A|c > 1/2])] > 1/2$ and takes action 1. Conversely, if $c_1 = \underline{c}$ and $c_2 = \overline{c}$, none of the thresholds is reached. The individual has a posterior belief $\Pr[A|c_1 < 1/2, c_2 < x^*(\Pr[A|c < 1/2])] < 1/2$ and takes action 0. From the perspective of an outside observer, the first piece of evidence acts as a reference point and conditions how further information is interpreted. In this particular case, a high first signal eventually leads to the action optimal in state A and a low first signal eventually leads to the action optimal in B. Naturally, the ordering of signals can never affect beliefs or actions in a model where the exact signals $\underline{c}$ and $\overline{c}$ are processed. The result is consistent with experimental evidence according to which first impressions, acting as an anchor, matter. Similar observations have been made for the cases of employment interview (Dougherty et al. (1994)), medical diagnosis (Elstein et al.(1978)) and judicial reasoning (Pennington and Hastie (1993)) among others.

In a similar vein and from the perspective of an outsider, subjects will appear *stubborn*. As they build up their confidence on one state, they become more and more likely to interpret ambiguous evidence as support for their beliefs. At the same time, strong contradictory evidence reverses their belief more dramatically. More generally, this theory supports the idea that people develop habits that are difficult to change and that people are less likely to change their mind with age.

Polarization of opinions

Individuals who exhibit confirmatory biases may interpret the same information in opposite ways. The result is known as the *polarization effect.* More precisely, this event occurs when mixed evidence is given to subjects whose existing views lie on both sides of the evidence. Their beliefs may then move farther apart. In an early work, Lord et al. (1979) presented a few studies on the deterrent effect of death penalty to a pool of subjects. When asked about the merits of death penalty, people who were initially in favor of (respectively against) capital punishment were more in favor (respectively against) after reading the studies (see also Darley and Gross (1983) and Plous (1991)).

The literature explains this effect in terms of cognitive biases and non-bayesian information processing. In particular, it is argued that individuals focus attention on the elements that support their original beliefs and (consciously or unconsciously) neglect the elements that contradict them. Our analysis suggests that attentional deficits and non-bayesian information processing need not be at the origin of this result. Instead, the decision-threshold mechanism can fully account for this behavior.

Implication 2 *(Polarization) Individuals with different priors who receive identical information may move their beliefs farther apart in opposite directions.*

Formally, consider two individuals, i and j, with prior beliefs p_i and p_j ($< p_i$). As shown in section 3, $x^*(p_i) < x^*(p_j)$. Suppose that 'mixed evidence' is released and both individuals perceive the same level of cell firing $c \in (x^*(p_i), x^*(p_j))$. Then, individual i will interpret the evidence in favor of A, update his belief to $p_i' > p_i$ and take a high action, whereas individual j will interpret the evidence in favor of B, update his belief to $p_j' < p_j$ and take a low action. Thus, the interpretative feature of the threshold mechanism generates a bias in the way the information is processed, which may lead to a radically different reading of identical evidence. Once again, this result cannot occur in a bayesian world if individuals receive and process the same signal.

Other biases: miss error rates in visual detection tasks

When subjects are asked to visually detect targets in a series of trials, it is shown that miss error rates are relatively higher if the frequency of targets is low (Mackworth (1970), Wolfe et al. (2007)). In other words, low probability events are incorrectly evaluated relatively more often than high probability events. The same observation has been made in studies on the detection of abnormalities in clinical radiology (Egglin and Feinstein, 1996). This effect is usually explained as a *criterion shift*, an idea consistent with our model. The decision threshold sets a criterion to interpret evidence, which is modulated by the likelihood of the event. It thus rationalizes the fact that individuals tend to see what they expect to see, and to miss what they do not expect to see.[16] In parallel, experimental

[16]We do not want to overemphasize this application since it would be also consistent with a standard

treatments in psychology show the prevalence of such a confirmatory bias when subjects are confronted with evidence that is uncertain, ambiguous or subjective. Also, varying the level of ambiguity affects the magnitude of the bias (Lord et al. (1979), Munro and Ditto (1997)).[17]

4.2 The role of payoffs on belief updating

The results of section 3 suggest a relation between beliefs and payoffs: payoffs influence decision-threshold which, in turn, affect the set of attainable ex-post beliefs. In other words, information interpretation and subsequent belief formation are shaped by the desirability of outcomes. This has a series of interesting implications.

Preferences and beliefs

The way information is interpreted depends on how the decision maker feels about alternatives, which itself affects the beliefs held ex-post. To illustrate this idea in the simplest possible terms, consider a population of agents identical in all respects except for π_A, the marginal disutility of taking a low action when the state is A. This difference can be subjective (fear of dying) or objective (likelihood of recovery from an injury). Assume that $\pi_A \in (\underline{\pi}, \overline{\pi})$. All individuals have the same prior p. They receive the same information and, for simplicity, experiment the same level of neuronal cell firing c. We obtain the following result.

Implication 3 *(Preferences shape beliefs) Individuals with identical priors but different utilities will hold systematically different posterior beliefs.*

From section 3, we know that $dx^*(\pi_A)/d\pi_A < 0$. Therefore, for a given cell-firing c, there exists a cutoff $\tilde{\pi}_A$ such that $c < x^*(\pi_A)$ for all $\pi_A < \tilde{\pi}_A$ and $c > x^*(\pi_A)$ for all $\pi_A > \tilde{\pi}_A$. Individuals in the first group take action 0 and revise their belief downwards ($\underline{p} < p$). Individuals in the second group take action 1 and revise their belief upwards ($\overline{p} > p$). In words, subjects sharing a prior and exposed to the same evidence may end up making different choices *and* holding different opinions. So, for instance, stronger individuals who are objectively less threatened by predators will decide more often to go

bayesian framework where attention is costly: as the likelihood of an event decreases, so does the expected benefit of trying to find an error. It would be difficult to discriminate between these two explanations. In fact, the criterion shift argument is in line with the findings in Appendix 5.

[17] This suggests that confirmatory biases are expected to play a stronger role when beliefs are based upon faith or tradition rather than on empirical evidence.

hunting than their weaker peers. This is rather obvious. More interestingly, they will also hold (on average) the belief that the environment is safer. This second conclusion cannot occur in the standard bayesian framework where c is processed. It also suggests an endogenous mechanism for what an outside observer may perceive as 'overconfidence' or 'ex-post rationalization:' individuals who go hunting report low danger whereas individual who stay in the cave report high danger. Again, these differences in opinions and excessive confidence are not based and irrational beliefs but on the economical threshold-based mechanism employed by the brain to process information.

Interestingly, there is often a subjective component in utility. Our mechanism may account for the fact that 'delusional' patients tend to report a high confidence in the states they consider undesirable. Those beliefs are classified as irrational in the psychology literature (Baron, 1988). According to our analysis, they may be internally consistent. For example, an individual who is terrified of dying (large perceived π_A) will set a low decision-threshold. As a consequence, he will interpret any minor symptom as a threat to his life. The analysis also suggests that, for those cases, treatments can be more effective if they target the perception of the loss rather than the perception of the likelihood of undesirable outcomes.

Non expected utility theory

The result emphasized above on 'preferences shaping beliefs' relates more generally to the idea that probabilities over outcomes and attitudes towards risks are interrelated. This issue has long been explored in the literature on non-expected utility, and several alternative utility representations have been suggested. Among the most notable, prospect theory proposes a probability weighting function to capture the tendency of decision-makers to over-react to small probability events and under-react to medium and large probability events (Kahneman and Tversky (1979), Prelec (1998)). Rank-dependent expected utility offers a representations in which outcomes are ranked and unlikely extreme outcomes are over-weighted (Quiggin (1982)). Finally, security-potential/aspiration (or SP/A) theory builds on the idea that fear and hope lead individuals to overweight the probabilities attached to the most undesirable and the most desirable events, respectively (see Lopes (1987) for the theory and Shefrin (2008) for applications to financial decision-making).[18]

[18] There exists a growing literature studying how the brain represents probabilities and preferences over outcomes in order to identify an appropriate model of decision-making under risk. See for instance Chew and Sagi (2008) for an axiomatization of the source preference hypothesis, and Hsu et al. (2005) and Chew et al. (2008) for neuroimaging studies identifying the brain regions that encode risk and uncertainty.

In those models, probabilities are considered objective but weighting functions introduce a subjective aspect as they may vary across individuals. Our model provides an alternative utility representation that accounts for similar effects.

Implication 4 *(Probability functions) An optimal decision-threshold mechanism generates payoff-dependent posterior beliefs.*

In our model, reports made on posterior beliefs are correlated with payoffs and behavior is not consistent with expected utility theory. In particular, the individual can be represented as an entity with generic utility function of the form:

$$\sum_{s=A,B} P_s(\pi_A, \pi_B)\, \pi_s\, \hat{l}(s, \gamma),$$

where $P_s(\pi_A, \pi_B)$ is the posterior belief and $\pi_s\, \hat{l}(s, \gamma)$ is the loss when action γ is taken in state s. The key issue is that the desirability of outcomes affects the decision-threshold, and therefore the choices *and* posterior beliefs. Perhaps the main difference between our model and the general approach followed in decision theory is that we do not explicitly consider a distinction between objective and subjective probabilities in the brain. Instead, the individual forms an opinion based on the stimuli received by the sensory system and on the report sent to the decision system. In particular, if the paradigm is uncommon (as in the Allais paradox for example) or presented in ambiguous terms, the decision-maker may form subjective probabilities and set decision-thresholds accordingly.

4.3 Elimination strategy in complex choices

The literature in social psychology has long emphasized the apparent inability of individuals to think through complex decisions with many alternatives (Payne (1982), Timmermans (1993), Hauser and Vermerfelt (1990)). To deal with these problems, individuals typically focus on a few salient options and completely neglect the rest. This is sometimes referred as an *elimination strategy*.

A three-state extension of our previous analysis can help understand this problem better. Formally, suppose that $S = \{A, O, B\}$, $\Gamma = [0, 1]$ and denote by p_S the prior probability of state S. Also, $\tilde{l}(\gamma, A) = l(\gamma - 1)$, $\tilde{l}(\gamma, O) = l(\gamma - \frac{1}{2})$ and $\tilde{l}(\gamma, B) = l(\gamma - 0)$ with $l(z) = -|z|$. Finally, when $S = O$, the probability of a cell firing level c is $f(c|O)$, with $\frac{d}{dc}\frac{f(c|B)}{f(c|O)} < 0$ and $\frac{d}{dc}\frac{f(c|O)}{f(c|A)} < 0$ for all c. In words, 'low', 'intermediate' and 'high' cell firing is imperfect evidence of states B, O and A respectively.

Given a linear loss function, a straightforward extension of the argument in Lemma 1 implies that the relevant action space is $\tilde{\gamma} \in \{0, \frac{1}{2}, 1\}$. A cognitive process needs to set only two decision-thresholds, which we denote $\underline{x}$ and $\overline{x}$, to discriminate optimally between these three alternatives.[19] The constrained optimal strategy of an affective process which can only set *one* decision-threshold is described below.

Implication 5 *(Elimination) The affective process discriminates optimally between the two relevant actions that are a priori most likely and fully disregards the third one. The utility loss relative to the cognitive process is greatest when all states are equally likely and smallest when one of the states is highly unlikely.*

An affective process necessarily results in some utility loss, as it can only discriminate between two actions. The issue is to determine where the threshold should be set and which action should be ignored. One could think that, even if an action is left out, it will still affect how the individual discriminates between the other two. As shown in appendix 6, this intuition is incorrect. Instead, the affective process sacrifices the action which is optimal in the state most unlikely to occur (e.g., action 1 if p_A is low relative to p_O and p_B), and then discriminates *optimally* between the other two (e.g., sets threshold $\underline{x}$ to perfectly differentiate between actions 0 and 1/2). The result is very much in line with the elimination strategy discussed above. States (and options) are categorized by relevance, which is a function of how probable they are. The least relevant option is fully disregarded and the most relevant ones are evaluated optimally within the reduced set. When an option is ignored then, for the purpose of the choice to be made, it is as if it did not even exist. In other words, the heuristic of the elimination strategy is proved to be optimal given the processing constraints. An immediate implication is that the cost of discriminating between only two actions will be a function of how likely the third one is.

5 Relation to neuroscience theories of decision-making

Two leading theories in neuroscience, somatic marker and cognitive control, argue that optimal decision-making is reached via a biasing process: information is interpreted in such a way that some actions are favored. In parallel, our model suggests that decision-thresholds affect the updating of beliefs and therefore the choices eventually made. We now investigate the relationship between those arguments.

[19] The actions selected are 0 if $c < \underline{x}$, 1/2 if $c \in [\underline{x}, \overline{x}]$ and 1 if $c > \overline{x}$

The Somatic Marker - Damasio (1994)

According to the Somatic Marker Theory (SMT - Damasio, 1994), decision-making is influenced by marker signals that arise in bio-regulatory processes. An emotion is a collection of changes that are triggered by a brain system that responds to perceptions. At the brain level, responses lead to the release of neurotransmitters that modulate neuronal activation. Emotions are induced by primary and secondary inducers. When making a decision, strong somatic markers are reinforced and weak ones are eliminated, generating a biasing mechanism. In our model, there is one primary inducer, c, and several secondary inducers, p and $\mathcal{L}$, which are recalled at the time of the decision. Primary and secondary inducers trigger an emotion that reflects how the evidence c should be interpreted in the face of p and $\mathcal{L}$, that is, the threshold x that leads to the interpretation $c > x$ or $c < x$. Thus, within this framework, an emotion is what makes the individual become aware of how he feels about the environment $\mathcal{L}$, what he thinks about it p, and how he should interpret new evidence c. Recognizing the parallel between our model and the SMT, we can derive a series of interesting implications.

First, the results in section 3 suggest that an emotional individual is likely to develop beliefs that are maintained and unlikely to develop beliefs that are abandoned. More precisely and compared to an individual with an abnormal activation of somatic signals, an emotional individual will modulate thresholds in a way that he is likely to stick to the same action as his prior intention was. This is true in simple (Proposition 1) and complex (Proposition 2) settings. Furthermore, when the threshold is surpassed, the belief is again updated upwards and the next threshold is decreased even further, resulting in a snowball effect (Proposition 3). The logic of the model is remarkably close to the informal argument of the SMT: "[P]re-existing somatic states influence the *threshold of neuronal cell firing* in trigger structures (e.g., VM cortex) so that subsequent somatic states from thoughts (secondary inducers) are triggered more or less easily. [...] While pre-existing negative somatic states *reinforce subsequent negative states*, they may impede the effectiveness of positive ones. Similarly, pre-existing positive states reinforce positive states, but they may impede negative ones" (Bechara and Damasio (2005, p. 363-4), italics added).

Second and related, threshold modulation in that particular direction improves decision-making. Therefore, if emotions are responsible for these particular threshold changes, then it is indeed the case that emotions help the optimal selection of actions. Again, the informal argument of the SMT follow those lines: "These somatic states are indeed beneficial, because they consciously or non-consciously *bias the decision in an advantageous* manner"

(Bechara and Damasio (2005, p. 351), italics added).

Finally, the results obtained in Appendix 5 suggest that the inability to modulate thresholds results in poorer choices when the situation is subtle (e.g., an action that may cause a moral harm on someone) than when the situation is straightforward to evaluate (e.g., an action that may cause a physical harm on someone). This hypothesis is largely favored in the SMT. Based on research with brain lesion patients, studies demonstrate that impairments in the emotional system have more dramatic effects in choices related to abstract or exceptional situations than in choices related to concrete or common situations (Anderson et al., 1999).

Cognitive Control - Miller and Cohen (2001)

Cognitive control is used to describe a collection of brain processes whose role is to guide thought and behavior in accordance with internally generated goals or plans. There is evidence that the control system is housed in the most anterior portion of the brain, the prefrontal cortex (PFC). It acts as a supervisory system, which can override automatic responses in favor of scheduling behavior on the basis of plans or intentions (Schneider and Shiffrin (1977), Norman and Shallice (1986)). Miller and Cohen (2001) argue that cognitive control is the primary function of the PFC (see also Ridderinkhof et al. (2004) for a review). Signals from the external world enter the sensory system, the information is passed to the motor system and actions are implemented. The prefrontal cortex performs cognitive control relying on a top-down processing that maps sensory inputs and thoughts into actions.

Making an optimal decision conditional on the fundamentals, p and $\mathcal{L}$, and the new information, c, can be interpreted as an internally generated goal. Setting x is a way to achieve it. Under this interpretation of the theory, an automatic process would be a process that uses a non-optimized, fixed threshold w (bottom-up processing along hardwire pathways). Whenever w is suboptimal and conflicts with the goal, executive functions are engaged to change this threshold into x (top-down processing performed by the PFC).

According to Miller and Cohen (2001), processing is competitive and different pathways carry different information and compete for the expression of behavior. Inputs may be ambiguous and it is necessary to tilt the competition in favor of certain selected features. In our model, setting a threshold reinforces a particular type of stimuli, which constitutes the biasing effect in the cognitive control theory.

6 Conclusion

Building theoretical models of brain processes is an important step both for economics and neuroscience. For economics, incorporating *physiological costs and constraints* in the capacity of individuals to evaluate situations, process information and reach conclusions has two advantages. First, it provides guidelines regarding the plausibility of different assumptions when we try and model bounded rationality. Second, it provides micro-microfoundations for some well-documented biases in choices (see Brocas and Carrillo (2008a) for a more detailed exposition of these arguments).[20] For neuroscience, formal models of the brain can provide testable implications about the functionality of different brain systems.

This paper has taken one step in that direction. We have provided a theoretical framework to study information processing in the brain. We have then used this framework to predict decisions in behaviorally relevant environments. Last, we have analyzed several implications of our theory and shown that our predictions are consistent with some anomalies and biases documented in psychology. Interestingly, those biases all originate in the same physiological constraints, and are thus likely to be observed in conjunction.

Our methodology can be extended in a number of directions. First, we do not account for the temporal aspects of information processing. An important literature documents correlations between reaction times, experimental conditions and decisions (see Ratcliff (1978) and the related diffusion model literature, and Schall (2003) for a review of recent findings on reaction times). Such findings may be incorporated to model the mechanism through which cell-firing reaches a particular threshold to trigger an action. So far in our model, it is simply assumed the threshold is reached. Second, we could analyze situations where individuals learn also from the choices made. In particular, we may want to compare the difference in beliefs before and after a decision is made and a stochastic outcome is realized. This could shed light on the reaction to expected and unexpected events. Last, the implications of our theory suggest that different individuals may process information differently. A natural extension is to analyze how individuals take decisions in strategic settings. For instance, we could measure the impact of confirmatory biases on decisions when agents engage in synergistic or competitive activities.

[20]One could draw a parallel with the theory of organizations, where a more accurate modelling of firm constraints (agency problems, restricted information channels, limited resource allocation) has helped understanding organizational choices.

Appendix

Appendix 1. Proof of Proposition 1.

Assume that the parameters of the model are such that $\frac{f(0|B)}{f(0|A)} > \frac{p}{1-p}\frac{\pi_A}{\pi_B} > \frac{f(1|B)}{f(1|A)}$.[21] Taking the first-order condition in (5), we find that $x^*(p)$ satisfies (6). Given **(A1)**, $x^*(p)$ is unique and the local second-order condition is satisfied:

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{x^*} = -p\, f'(x^*|A)\pi_A(l(0)-l(1)) + (1-p)f'(x^*|B)\pi_B(l(0)-l(1))$$
$$= (1-p)\pi_B\, f(x^*|A)(l(0)-l(1))\left(\tfrac{f(x^*|B)}{f(x^*|A)}\right)' < 0.$$

$dx^*/dp < 0$ is immediate from **(A1)**. Finally, we can ex-post that it is optimal to select $\gamma^* = 1$ when $c > x^*$ and $\gamma^* = 0$ when $c < x^*$. We have:

$\Pr(A|c > x^*) > \frac{\pi_B}{\pi_A+\pi_B} \Leftrightarrow p\pi_A(1-F(x^*|A)) > (1-p)\pi_B(1-F(x^*|B)) \Leftrightarrow \frac{1-F(x^*|A)}{f(x^*|A)} > \frac{1-F(x^*|B)}{f(x^*|B)}$

$\Pr(A|c < x^*) < \frac{\pi_B}{\pi_A+\pi_B} \Leftrightarrow p\,\pi_A\, F(x^*|A) > (1-p)\pi_B F(x^*|B) \Leftrightarrow \frac{F(x^*|A)}{f(x^*|A)} < \frac{F(x^*|B)}{f(x^*|B)}$

Both inequalities are satisfied given **(A1)**. This completes the proof.

Appendix 2. Proof of Proposition 2.

Given (9), we can rewrite the value function as:

$$V(x) = p(1-F(x|A))\pi_A l(1-\gamma^{**}(\overline{p}(x))) + (1-p)(1-F(x|B))\pi_B l(\gamma^{**}(\overline{p}(x))) + p\,F(x|A)\,\pi_A l(1-\gamma^{**}(\underline{p}(x))) + (1-p)F(x|B)\,\pi_B l(\gamma^{**}(\underline{p}(x))) \quad (15)$$

The optimal threshold maximizes (15) given (7) and (8). The first-order condition is:

$$\left.\frac{\partial V(x)}{\partial x}\right|_{x=x^{**}} = 0 \;\Rightarrow\; \frac{f(x^{**}|B)}{f(x^{**}|A)} = \frac{p}{1-p}\,\frac{\pi_A}{\pi_B}\,\frac{l(1-\gamma^{**}(\overline{p}(x^{**}))) - l(1-\gamma^{**}(\underline{p}(x^{**})))}{l(\gamma^{**}(\underline{p}(x^{**}))) - l(\gamma^{**}(\overline{p}(x^{**})))} \quad (16)$$

From (16) and using (7) and (8), we get:

$$\left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x=x^{**}} = -\frac{\pi_B}{p}\left[f(x^{**}|B)\Big(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\overline{p}))\Big) + F(x^{**}|B)l'(\gamma^{**}(\underline{p}))\left.\frac{d\gamma^{**}(\underline{p})}{dx}\right|_{x^{**}} + (1-F(x^{**}|B))l'(\gamma^{**}(\overline{p}))\left.\frac{d\gamma^{**}(\overline{p})}{dx}\right|_{x^{**}}\right]$$
$$= -\frac{\pi_B}{p}\times\frac{d}{dx}\Big[F(x|B)l(\gamma^{**}(\underline{p})) + (1-F(x|B))l(\gamma^{**}(\overline{p}))\Big]_{x=x^{**}}$$

[21] This condition ensures that the optimal solution is interior.

Similarly,

$$\left.\frac{\partial^2 V(x;p)}{\partial x^2}\right|_{x=x^{**}} = (1-p)\pi_B \frac{\left(\frac{f(x|B)}{f(x|A)}\right)'}{\frac{f(x|B)}{f(x|A)}} \Bigg[f(x|B)\Big(l(\gamma^{**}(\underline{p})) - l(\gamma^{**}(\overline{p}))\Big)$$

$$+F(x|B)l'(\gamma^{**}(\underline{p}))\frac{d\gamma^{**}(\underline{p})}{dx}\frac{\frac{f(x|B)}{f(x|A)}}{\left(\frac{f(x|B)}{f(x|A)}\right)'}\frac{\left(\frac{F(x|B)}{F(x|A)}\right)'}{\frac{F(x|B)}{F(x|A)}}$$

$$\left. + (1-F(x|B))l'(\gamma^{**}(\overline{p}))\frac{d\gamma^{**}(\overline{p})}{dx}\frac{\frac{f(x|B)}{f(x|A)}}{\left(\frac{f(x|B)}{f(x|A)}\right)'}\frac{\left(\frac{1-F(x|B)}{1-F(x|A)}\right)'}{\frac{1-F(x|B)}{1-F(x|A)}} \Bigg]\right|_{x=x^{**}}$$

By **(A2)**, $\frac{\frac{f(x|B)}{f(x|A)}}{\left(\frac{f(x|B)}{f(x|A)}\right)'}\frac{\left(\frac{F(x|B)}{F(x|A)}\right)'}{\frac{F(x|B)}{F(x|A)}} \leqslant 1$ and $\frac{\frac{f(x|B)}{f(x|A)}}{\left(\frac{f(x|B)}{f(x|A)}\right)'}\frac{\left(\frac{1-F(x|B)}{1-F(x|A)}\right)'}{\frac{1-F(x|B)}{1-F(x|A)}} \leqslant 1$. Therefore, $\left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x=x^{**}} < 0 \Rightarrow \left.\frac{\partial^2 V(x;p)}{\partial x^2}\right|_{x=x^{**}} < 0$ and the proposition follows.

Characterization of the equilibrium with quadratic loss. Suppose that $l(z) = \alpha - \beta\, z^2$ with $\beta > 0$ and $\pi_A = \pi_B = 1$. Under this restriction, (7) and (8) become:

$$\gamma^{**}(\overline{p}(x)) = \overline{p}(x) \quad \text{and} \quad \gamma^{**}(\underline{p}(x)) = \underline{p}(x)$$

Therefore, (10) has the following simple expression:

$$\frac{f(x^{**}|B)}{f(x^{**}|A)} = \frac{p}{1-p}\,\frac{(1-\overline{p}(x^{**})) + (1-\underline{p}(x^{**}))}{\overline{p}(x^{**}) + \underline{p}(x^{**})} \tag{17}$$

Let $P \equiv \frac{p}{1-p}$. The F.O.C. (17) can be rewritten as:

$$\frac{1-p}{p^2}\left.\frac{\partial V(x;p)}{\partial x}\right|_{x^{**}} = k(x^{**}, P)$$

where $k(x,P) \equiv \left(\frac{1-F(x|A)}{(1-F(x|A))P+(1-F(x|B))} - \frac{F(x|A)}{F(x|A)\,P+F(x|B)}\right)\Big[f(x|B)\Big(\frac{1-F(x|A)}{(1-F(x|A))P+(1-F(x|B))} + \frac{F(x|A)}{F(x|A)\,P+F(x|B)}\Big) - f(x|A)\Big(\frac{1-F(x|B)}{(1-F(x|A))P+(1-F(x|B))} + \frac{F(x|B)}{F(x|A)\,P+F(x|B)}\Big)\Big]$. Differentiating the F.O.C., we get

$$\left.\frac{\partial^2 V(x;p)}{\partial x \partial p}\right|_{x^{**}} \propto \frac{\partial k(x^{**}, P)}{\partial P}$$

After some tedious algebra, and using (17), we get:

$$\frac{\partial k(x^{**},P)}{\partial P} = -f(x^{**}|A)\frac{\left(\frac{F(x^{**}|B)}{F(x^{**}|A)} - \frac{1-F(x^{**}|B)}{1-F(x^{**}|A)}\right)^3}{\left(2P + \frac{F(x^{**}|B)}{F(x^{**}|A)} + \frac{1-F(x^{**}|B)}{1-F(x^{**}|A)}\right)\left(P + \frac{F(x^{**}|B)}{F(x^{**}|A)}\right)^2\left(P + \frac{1-F(x^{**}|B)}{1-F(x^{**}|A)}\right)^2} < 0$$

which guarantees that $dx^{**}/dp < 0$ is satisfied in every locally optimal threshold.

Analytical example in the linear and quadratic cases. Let $F(c|A) = c^2$ and $F(c|B) = c$. From (6) and (17) and after some algebra, the optimal thresholds with linear ($l(z) = -|z|$) and quadratic ($l(z) = -z^2$) payoffs are respectively:

$$x^*(p) = \tfrac{1-p}{2p} \quad \text{and} \quad x^{**}(p) = \tfrac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{1+p}}$$

where x^* and x^{**} are interior if $p > 1/3$. In this example, the optimal threshold is always less extreme with quadratic than with linear payoffs: $x^* \gtreqless x^{**} \gtreqless 1/3$ for all $p \lesseqgtr 3/5$.

Appendix 3. Proof of Proposition 3.

Taking the first-order condition in (13) and applying the envelope theorem, we get:

$$\left.\frac{\partial W(y;p)}{\partial y}\right|_{y=y^*} = 0 \quad \Rightarrow \quad \frac{f_1(y^*|B)}{f_1(y^*|A)} = \frac{p}{1-p}\,\frac{F_2(x^*(\underline{p}(y^*))|A) - F_2(x^*(\overline{p}(y^*))|A)}{F_2(x^*(\underline{p}(y^*))|B) - F_2(x^*(\overline{p}(y^*))|B)}\,\frac{\pi_A}{\pi_B}$$

The rest of the proof follows the exact same steps as the proof of Proposition 2 and is therefore omitted for the sake of brevity.

Equilibrium with second stage linear densities. Let $g_2(c) = 2c$ and $f_2(c) = 2(1-c)$, $\pi_A = \pi_B = 1$, and let us and keep a general formulation for the first stage cell firing densities. After some algebra, the first-order condition (14) can be rewritten as:

$$\frac{f_1(y^*|B)}{f_1(y^*|A)} = \frac{p}{1-p}\,\frac{(1-\overline{p}(y^*)) + (1-\underline{p}(y^*))}{\overline{p}(y^*) + \underline{p}(y^*)}$$

which is exactly the same expression as (17), and the result follows.

Example 1, continued. Suppose $l(0) = 1$, $l(1) = 0$ and $p = 1/2$. It follows that $y^* = \frac{1}{2}$, $\overline{p}(\frac{1}{2}) = \frac{3}{4}$, $x^*(\overline{p}(\frac{1}{2})) = \frac{1}{4}$, $\underline{p}(\frac{1}{2}) = \frac{1}{4}$, and $x^*(\underline{p}(\frac{1}{2})) = \frac{3}{4}$. A continuum of thresholds in stage 1 is formally equivalent to observing the exact cell firing c_1. Given $p(A|c_1) = c_1$, the expected utility if the cognitive process is employed is:

$$\tilde{W} = p\int_0^1 \Pr(c_1|A)\Big[1-F(x^*(p(A|c_1))|A)\Big]dc_1 + (1-p)\int_0^1 \Pr(c_1|B)\Big[F(x^*(p(A|c_1))|B)\Big]dc_1 = \frac{5}{6}$$

If, instead, the affective process is employed, the individual at stage 1 only learns whether $c_1 \gtrless y^*$ $(= 1/2)$. Following (13), his expected utility is:

$$\begin{aligned} W \;=\; &\Pr(A)\Big[\Pr(c_1 > \tfrac{1}{2}\,|\,A)\,\Pr(c_1 > \tfrac{1}{4}\,|\,A) + \Pr(c_1 < \tfrac{1}{2}\,|\,A)\Pr(c_1 > \tfrac{3}{4}\,|\,A)\Big] \\ &+ \Pr(B)\Big[\Pr(c_1 < \tfrac{1}{2}\,|\,B)\,\Pr(c_1 < \tfrac{3}{4}\,|\,B) + \Pr(c_1 > \tfrac{1}{2}\,|\,B)\Pr(c_1 < \tfrac{1}{4}\,|\,B)\Big] \;=\; \frac{13}{16} \;<\; \tilde{W} \end{aligned}$$

In this example, the expected utility loss of using the affective process is only 2.5%.

Appendix 4. Optimal threshold with two actions and a continuum of states.

Let $S = [0,1]$ and $\Gamma = \{0,1\}$. In our example, $s \in S$ captures the proportion of predators in the neighborhood. The individual decides between hunting ($\gamma = 0$) and staying in the cave ($\gamma = 1$). We order the states by the increasing degree of danger, from safest ($s = 0$) to most dangerous ($s = 1$). Payoffs are $\tilde{l}(1,s) = \pi_s\, l(1-s)$ and $\tilde{l}(0,s) = \pi_s\, l(-s)$, where $l(z) = l(-z)$ for all z and $l'(z) < 0$ for all $z > 0$. The probability of cell firing level c given state s is now $f(c \,|\, s)$. The generalization of MLRP to the continuous case is:

Assumption 1' (continuous MLRP) $\dfrac{d}{dc}\left(\dfrac{f_s(c \,|\, s)}{f(c \,|\, s)}\right) \geqslant 0$ for all c and s. **(A1')**

The individual believes that the state is s with probability $p(s)$. The expected payoff (3) is generalized as $L(p(s),\gamma) = \int_0^1 p(s)\,\pi_s\, l(\gamma - s)ds$ and the optimal action is:

$$\hat{\gamma} = 1 \text{ if } \int_0^1 p(s)\,\pi_s \Big(l(1-s) - l(s)\Big)ds > 0 \quad \text{and} \quad \hat{\gamma} = 0 \text{ if } \int_0^1 p(s)\,\pi_s \Big(l(1-s) - l(s)\Big)ds < 0$$

To simplify the analysis, assume that $\pi_s = \pi_l$ for all states $s < 1/2$ and $\pi_s = \pi_u$ for all states $s \geqslant 1/2$. The value function is:

$$\begin{aligned} V(x;p(s)) &= \Pr(c > x)\, L(p(s \,|\, c > x), 1) + \Pr(c < x)\, L(p(s \,|\, c < x), 0) \\ &= \int_0^1 p(s)\pi_s\Big((1 - F(c \,|\, s))l(1-s) + F(c \,|\, s)l(s)\Big)ds \end{aligned} \tag{18}$$

Denote by $\hat{x}(p(s)) = \arg\max_x \; V(x;p(s))$. Taking the F.O.C. in (18), we obtain:

$$-\int_0^1 p(s)\,\pi_s\, f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds = 0 \tag{19}$$

The local S.O.C. is:

$$\begin{aligned} \left.\frac{\partial^2 V}{\partial x^2}\right|_{\hat{x}} &= -\int_0^1 p(s)\,\pi_s\, f_x(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds \\ &= \int_0^1 \left(-\frac{f_x(\hat{x} \,|\, s)}{f(\hat{x} \,|\, s)}\right) p(s)\,\pi_s\, f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds \end{aligned}$$

Let $h(s) \equiv -\dfrac{f_x(\hat{x} \,|\, s)}{f(\hat{x} \,|\, s)}$. By **(A1')**, $h'(s) \leqslant 0$. We can then rewrite the local S.O.C. as:

$$\begin{aligned} \left.\frac{\partial^2 V}{\partial x^2}\right|_{\hat{x}} &= \int_0^{1/2} h(s)\,p(s)\,\pi_s\, f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds + \int_{1/2}^1 h(s)\,p(s)\,\pi_s\; f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds \\ &< h(1/2)\left[\int_0^{1/2} p(s)\,\pi_s\, f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds + \int_{1/2}^1 p(s)\,\pi_s\, f(\hat{x} \,|\, s)\Big(l(1-s) - l(s)\Big)ds\right] = 0 \end{aligned}$$

which means that the threshold $\hat{x}(p(s))$ defined by (19) is indeed a unique maximum.

Suppose now that $\left(\frac{p(s)}{q(s)}\right)' \leqslant 0$, then:

$$\begin{aligned} \left.\frac{\partial V(x;p(s))}{\partial x}\right|_{\hat{x}(p(s))} &= -\int_0^1 \left(\frac{p(s)}{q(s)}\right) q(s)\,\pi_s\, f(\hat{x}(p(s))\,|\,s)\Big(l(1-s)-l(s)\Big)ds \\ &> -\left(\frac{p(1/2)}{q(1/2)}\right)\int_0^1 q(s)\,\pi_s\, f(\hat{x}(p(s))\,|\,s)\Big(l(1-s)-l(s)\Big)ds \end{aligned}$$

Therefore,

$$\left.\frac{\partial V(x;p(s))}{\partial x}\right|_{\hat{x}(p(s))} = 0 \;>\; \left(\frac{p(1/2)}{q(1/2)}\right)\left.\frac{\partial V(x;q(s))}{\partial x}\right|_{\hat{x}(p(s))} \;\Rightarrow\; \hat{x}(p(s)) > \hat{x}(q(s))$$

In words, if one individual puts more weight in higher states than another one in a MLRP sense, $\left(\frac{q(s)}{p(s)}\right)' \geqslant 0$, then he also sets a lower threshold. This property is simply a generalization of the comparative statics on p to the case of a continuous distribution of beliefs.

Finally, we need to check that it is indeed optimal to choose $\hat{\gamma} = 1$ when $c > \hat{x}$ and $\hat{\gamma} = 0$ when $c < \hat{x}$. Let $\mathcal{J}(x) \equiv L(p(s\,|\,c = x), 1) - L(p(s\,|\,c = x), 0)$, also $p(s\,|\,c = x) \equiv j(s\,|\,x) = \frac{p(s)f(x\,|\,s)}{\int_0^1 p(s)f(x\,|\,s)ds}$ and $J(s\,|\,x) = \int_0^s j(\tilde{s}\,|\,x)d\tilde{s}$. We use the fact that $\pi_s = \pi_l$ for all $s < 1/2$ and $\pi_s = \pi_u$ for all $s \geqslant 1/2$. Integrating by parts:

$$\begin{aligned} \mathcal{J}(x) &= \int_0^1 j(s\,|\,x)\,\pi_s\,\Big(l(1-s)-l(s)\Big)ds \\ &= \pi_u\Big(l(0)-l(1)\Big) + \int_0^1 J(s\,|\,x)\,\pi_s\,\Big(l'(1-s)+l'(s)\Big)ds \end{aligned}$$

Therefore

$$\frac{d\mathcal{J}(x)}{dx} = \int_0^1 J_x(s\,|\,x)\,\pi_s\,\Big(l'(1-s)+l'(s)\Big)ds > 0$$

since, by **(A1')**, we know that $F_s(x\,|\,s) < 0$ and therefore $J_x(s\,|\,x) < 0$. From (19), $\mathcal{J}(\hat{x}) = 0$, so $\mathcal{J}(x) \gtreqless 0$ for all $x \gtreqless \hat{x}$. This also proves that, for the purpose of the action to be taken, it is equivalent to learn c or to learn whether c is greater or smaller than $\hat{x}$.

Last, setting $\hat{\mathcal{L}} = \frac{\pi_u}{\pi_l}$ and differentiating (19) with respect to $\hat{\mathcal{L}}$, we obtain:

$$\frac{1}{\pi_l}\left.\frac{\partial^2 V}{\partial x^2}\right|_{\hat{x}}\frac{\partial \hat{x}}{\partial \hat{\mathcal{L}}} - \int_{1/2}^1 p(s)\, f_x(\hat{x}\,|\,s)\Big(l(1-s)-l(s)\Big)ds = 0$$

which implies that $d\hat{x}/d\hat{\mathcal{L}} < 0$. Summing up, the conclusions stated in Proposition 1 (no loss of utility by setting only one optimal cutoff and comparative statics of optimal cutoff with respect to prior beliefs and cost of wrong actions) extend to the case of two actions and a continuum of states.

Appendix 5. Optimal thresholds in simple vs. complex activities.

Consider activities "α" and "β" such that low cell firing when $S = A$ or high cell firing when $S = B$ are uniformly less frequent in α-activities than in β-activities. Formally:

$$\frac{\partial}{\partial c}\left(\frac{f_\alpha(c|A)}{f_\beta(c|A)}\right) > 0 \quad \text{and} \quad \frac{\partial}{\partial c}\left(\frac{f_\alpha(c|B)}{f_\beta(c|B)}\right) < 0 \tag{20}$$

where $f_k(c|S)$ is the probability of cell firing c in situation $k \in \{\alpha, \beta\}$ given state S. The idea is simply that "neuronal mistakes", defined as low cell firing when $S = A$ or high cell firing when $S = B$, are uniformly less frequent in α-activities than in β-activities. In other words, α activities represent simple (concrete, common, temporally close) choices whereas β activities represent complex (abstract, exceptional, temporally distant) choices. For technical reasons, we also assume that **(A1)** is satisfied in both type of activities. $\left(\frac{f_\alpha(c|A)}{f_\beta(c|A)}\right)' > 0$, $\left(\frac{f_\beta(c|B)}{f_\beta(c|A)}\right)' < 0$, $\left(\frac{f_\alpha(c|B)}{f_\beta(c|B)}\right)' < 0$, $\left(\frac{f_\alpha(c|B)}{f_\alpha(c|A)}\right)' < 0 \;\Rightarrow\; \frac{f'_\alpha(c|A)}{f_\alpha(c|A)} > \frac{f'_\beta(c|A)}{f_\beta(c|A)} > \frac{f'_\beta(c|B)}{f_\beta(c|B)} > \frac{f'_\alpha(c|B)}{f_\alpha(c|B)}$. Now, suppose there exists $\hat{c} \in (0,1)$ such that $\frac{f_\alpha(\hat{c}|B)}{f_\alpha(\hat{c}|A)} = \frac{f_\beta(\hat{c}|B)}{f_\beta(\hat{c}|A)}$. Then,

$$\frac{d}{dc}\left[\frac{f_\alpha(c|B)}{f_\alpha(c|A)} - \frac{f_\beta(c|B)}{f_\beta(c|A)}\right]_{c=\hat{c}} = \frac{f_\alpha(\hat{c}|B)}{f_\alpha(\hat{c}|A)}\left(\frac{f'_\alpha(\hat{c}|B)}{f_\alpha(\hat{c}|B)} - \frac{f'_\alpha(\hat{c}|A)}{f_\alpha(\hat{c}|A)}\right) - \frac{f_\beta(\hat{c}|B)}{f_\beta(\hat{c}|A)}\left(\frac{f'_\beta(\hat{c}|B)}{f_\beta(\hat{c}|B)} - \frac{f'_\beta(\hat{c}|A)}{f_\beta(\hat{c}|A)}\right) < 0$$

so $\frac{f_\alpha(c|B)}{f_\alpha(c|A)}$ and $\frac{f_\beta(c|B)}{f_\beta(c|A)}$ cross at most once. Also, $\left(\frac{f_\alpha(c|A)}{f_\beta(c|A)}\right)' > 0$ and $\left(\frac{f_\alpha(c|B)}{f_\beta(c|B)}\right)' < 0 \;\Rightarrow\; \frac{f_\alpha(0|B)}{f_\alpha(0|A)} > \frac{f_\beta(0|B)}{f_\beta(0|A)}$ and $\frac{f_\alpha(1|B)}{f_\alpha(1|A)} < \frac{f_\beta(1|B)}{f_\beta(1|A)}$. Together with the previous result, it means that there exists $\hat{x} \in (0,1)$ such that $\frac{f_\alpha(x|B)}{f_\alpha(x|A)} \gtreqless \frac{f_\beta(x|B)}{f_\beta(x|A)}$ for all $x \lesseqgtr \hat{x}$. Denote by $x^*_k(p)$ the optimal threshold in activity k as a function of p. Given (6), there exists $\hat{p}$ such that $\frac{f_\alpha(x^*_\alpha(\hat{p})|B)}{f_\alpha(x^*_\alpha(\hat{p})|A)} = \frac{f_\beta(x^*_\beta(\hat{p})|B)}{f_\beta(x^*_\beta(\hat{p})|A)} = \frac{\hat{p}}{1-\hat{p}}$, that is, $x^*_\alpha(\hat{p}) = x^*_\beta(\hat{p}) = x^*(\hat{p}) \equiv \hat{x}$. For all $p \gtrless \hat{p}$, $\frac{f_\alpha(x^*_\alpha(p)|B)}{f_\alpha(x^*_\alpha(p)|A)} = \frac{f_\beta(x^*_\beta(p)|B)}{f_\beta(x^*_\beta(p)|A)} = \frac{p}{1-p} \;\Rightarrow\; x^*_\beta(p) \lesseqgtr x^*_\alpha(p) \lesseqgtr \hat{x}$. Overall, optimal thresholds are more sensitive to initial beliefs in complex than in simple activities.

Now, suppose $p < \hat{p}$. Then, $x^*_\alpha < x^*_\beta$. Given **(A1)**, $\frac{F_\alpha(x^*_\alpha|A)}{F_\alpha(x^*_\alpha|B)} < \frac{F_\alpha(x^*_\beta|A)}{F_\alpha(x^*_\beta|B)}$. Given (20), $\frac{F_\alpha(x^*_\beta|A)}{F_\alpha(x^*_\beta|B)} < \frac{F_\beta(x^*_\beta|A)}{F_\beta(x^*_\beta|B)}$. Finally, $\frac{F_\alpha(x^*_\alpha|A)}{F_\alpha(x^*_\alpha|B)} < \frac{F_\beta(x^*_\beta|A)}{F_\beta(x^*_\beta|B)} \;\Leftrightarrow\; \Pr_\beta[A|0] > \Pr_\alpha[A|0]$.

Analogously, if $p > \hat{p}$ then $x^*_\alpha > x^*_\beta$. Given **(A1)** and (20), $\frac{1-F_\beta(x^*_\beta|B)}{1-F_\beta(x^*_\beta|A)} > \frac{1-F_\beta(x^*_\alpha|B)}{1-F_\beta(x^*_\alpha|A)} > \frac{1-F_\alpha(x^*_\alpha|B)}{1-F_\alpha(x^*_\alpha|A)}$, and therefore $\Pr_\beta[B|1] > \Pr_\alpha[B|1]$. This proves that the individual is more likely to make mistakes in complex rather than simple activities.

Appendix 6. Proof of Implication 5.

Cognitive process. A necessary condition for cutoffs x_1 and x_2 ($> x_1$) to be optimal is $\tilde{\gamma} = 0$ if $c \in [0, x_1)$, $\tilde{\gamma} = \frac{1}{2}$ if $c \in [x_1, x_2]$ and $\tilde{\gamma} = 1$ if $c \in (x_2, 1]$. The value function is then:

$$\begin{aligned}
V(x_1, x_2) &= \Pr(c < x_1)L(p(\cdot \,|\, c < x_1), 0) + \Pr(c \in [x_1, x_2])L(\tfrac{1}{2}; p(\cdot \,|\, c \in [x_1, x_2])) \\
&\quad + \Pr(c > x_2)L(p(\cdot \,|\, c > x_2), 1) \\
&= -p_B\Big[(1 - F(x_2|B)) + \tfrac{1}{2}(F(x_2|B) - F(x_1|B))\Big] \qquad (21) \\
&\quad - p_O\Big[\frac{1}{2}(1 - F(x_2|O)) + \frac{1}{2}F(x_1|O)\Big] - p_A\Big[\tfrac{1}{2}(F(x_2|A) - F(x_1|A)) + F(x_1|A)\Big]
\end{aligned}$$

Taking F.O.C. in (21), we obtain $\underline{x}$ and $\overline{x}$. They solve:

$$\frac{f(\underline{x}|B)}{f(\underline{x}|A)} = \frac{p_A}{p_B} + \frac{p_O}{p_B}\frac{f(\underline{x}|O)}{f(\underline{x}|A)} \qquad \text{and} \qquad \frac{f(\overline{x}|B)}{f(\overline{x}|A)} = \frac{p_A}{p_B} - \frac{p_O}{p_B}\frac{f(\overline{x}|O)}{f(\overline{x}|A)}$$

Notice that $\underline{x} < \overline{x}$ for all $p_A, p_O, p_B \in (0,1)^3$ and $\underline{x} = x^* = \overline{x}$ when $p_O = 0$.We also have $\frac{\partial^2 V(x_1,x_2)}{\partial x_1^2}\Big|_{\underline{x}} = -\frac{1}{2}p_O f(\underline{x}|B)\left(\frac{f(\underline{x}|O)}{f(\underline{x}|B)}\right)' - \frac{1}{2}p_A f(\underline{x}|B)\left(\frac{f(\underline{x}|A)}{f(\underline{x}|B)}\right)' < 0$, $\frac{\partial^2 V(x_1,x_2)}{\partial x_2^2}\Big|_{\overline{x}} = \frac{1}{2}p_O f(\overline{x}|A)\left(\frac{f(\overline{x}|O)}{f(\overline{x}|A)}\right)' + \frac{1}{2}p_B f(\overline{x}|A)\left(\frac{f(\overline{x}|B)}{f(\overline{x}|A)}\right)' < 0$, and $\frac{\partial^2 V(x_1,x_2)}{\partial x_1 \partial x_2} = 0$. Therefore $\underline{x}$ and $\overline{x}$ are maxima. Last, it can be easily checked that $\Pr(S \,|\, c \in \mathcal{Y})$, $S \in \{A, O, B\}$ are such that $\tilde{\gamma} = 0$ if $\mathcal{Y} = [0, \underline{x})$, $\tilde{\gamma} = \frac{1}{2}$ if $\mathcal{Y} = [\underline{x}, \overline{x}]$, and $\tilde{\gamma} = 1$ if $\mathcal{Y} = (\overline{x}, 1]$ are indeed optimal.

Affective process. Let $\breve{x}$ be the cutoff that solves:

$$\frac{f(\breve{x}|B)}{f(\breve{x}|A)} = \frac{p_A}{p_B}$$

It is immediate to see that $\breve{x} \in (\underline{x}, \overline{x})$. The three candidates for optimal cutoffs are:

$$\begin{cases}
x_a \ \text{ so that } \ \tilde{\gamma} = 0 & \text{if } \ c < x_a \ \text{ and } \ \tilde{\gamma} = 1 \ \ \text{if } \ c > x_a \\
x_b \ \text{ so that } \ \tilde{\gamma} = 0 & \text{if } \ c < x_b \ \text{ and } \ \tilde{\gamma} = 1/2 \ \ \text{if } \ c > x_b \\
x_c \ \text{ so that } \ \tilde{\gamma} = 1/2 & \text{if } \ c < x_c \ \text{ and } \ \tilde{\gamma} = 1 \ \ \text{if } \ c > x_c
\end{cases}$$

These cutoffs are formally defined by:

$$\begin{cases}
x_a = \arg\max_x \ V^a(x) \equiv \Pr(c < x)\, L(0; p(\cdot\,|c < x)) + \Pr(c > x)\, L(1; p(\cdot\,|c < x)) \\
x_b = \arg\max_x \ V^b(x) \equiv \Pr(c < x)\, L(0; p(\cdot\,|c < x)) + \Pr(c > x)\, L(\tfrac{1}{2}; p(\cdot\,|c < x)) \\
x_c = \arg\max_x \ V^c(x) \equiv \Pr(c < x)\, L(\tfrac{1}{2}; p(\cdot\,|c < x)) + \Pr(c > x)\, L(1; p(\cdot\,|c < x))
\end{cases}$$

It is straightforward to check that $x_a = \breve{x}$, $x_b = \underline{x}$, $x_c = \overline{x}$. Now, fix p_O. Differentiating each first-order condition with respect to p_B, we get:

$$\frac{dx_a}{dp_B} > 0, \quad \frac{dx_b}{dp_B} > 0, \quad \frac{dx_c}{dp_B} > 0$$

Furthermore:

$$\frac{dV^a(x^a)}{dp_B} = F(x^a|B) + F(x^a|A) - 1 \gtrless 0, \quad \frac{d^2V^a(x^a)}{dp_B^2} = \Big[f(x^a|B) + f(x^a|A)\Big]\frac{dx_a}{dp_B} \geqslant 0,$$

$$\frac{dV^b(x^b)}{dp_B} = \frac{F(x^b|B) + F(x^b|A)}{2} \geqslant 0, \quad \frac{dV^c(x^c)}{dp_B} = \frac{F(x^c|B) + F(x^c|A)}{2} - 1 \leqslant 0$$

Also, $\lim_{p_B \to 0} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \to 0} V^c(x^c)$ and $\lim_{p_B \to 1-p_O} V^a(x^a) = -\frac{p_O}{2} < \lim_{p_B \to 1-p_O} V^b(x^b)$.

Combining these results, we have that there exist p^* such that x^c dominates x^b if $p_B < p^*$ and x^b dominates x^c if $p_B > p^*$. Also, there exist p^{**} and p^{***} such that x^c dominates x^a if $p_B < p^{**}$ and x^b dominates x^a if $p_B > p^{***}$. The ranking between p^*, p^{**} and p^{***} depend on the relative values of p_O and p_A.

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